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Cross- and delta-hedges: Regression- versus price-based hedge ratios

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Abstract

In implementing a variance-minimizing cross or delta hedge, the regression coefficient is often estimated using data from the past, but one could also use estimators that are suggested by the random-walk or unbiased-expectations models and require just a single price. We compare the performances of various hedge ratios for three-month currency exposures, and find that the price-based hedge ratios generally perform better than the regression-based ones. Specifically, all our regressions do systematically worse in the case of a delta hedge, and seem to beat the price-based hedge ratios only in the case of cross- or cross-and-delta problems where the two currencies are so distantly related – like, e.g., hedging ITL/USD using JPY/USD – that no risk manager would even consider them as hedges of each other. The poor performance of the regressions is all the more surprising as we correct the futures prices for errors-in-variables (synchronization noise, bid–ask bounce, and changing time to maturity).

The results are robust to observation frequency in the regressions, sample period, percentage vs dollar returns, and OLS versus IV. One reason why price-based methods do better is that they provide immediate adjustment to breaks in the data (like EMS realignments, which get incorporated into rolling regression coefficients only very slowly, as time elapses) or other events that change the relationship between the

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regressor and regressand. For cross or cross-and-delta hedges between European currencies, regressions also have difficulties in capturing cross-correlations between exchange rates. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

When hedging a contractual exposure in futures markets, one usually has to settle for a hedge instrument that fails to match the exposure in terms of expiry date (“delta hedge”), underlying asset (“cross hedge”), or both. The usual approach, since Johnson (1960) and Stein (1961), is to select the number of futures contracts that minimizes the conditional variance of the hedged position, and the resulting optimal hedge ratio is given by the slope coefficient of a conditional regression between the future spot rate that one is exposed to and the futures price that is being used as a hedge. In the literature, hedge ratios tend to be computed from past data (typically first differences), using unconditional techniques.² In this paper, we compare the performance of various regression-based hedge ratios to alternative estimators that are suggested by the random-walk or unbiased-expectation model and require just a single price. Unexpectedly, these price-based hedge ratios tend to dominate the former, even after taking care of the estimation problems that potentially plague regression-based hedge ratios.

To set the stage for a comparison of our procedures and findings to the work of others, we first briefly review the estimation issues. As Stoll and Whaley (1993) note, one source of problems when implementing a regression-based hedge strategy is data imperfections. For example, the spot and futures prices used in the regressions are often not fully synchronized because of reporting lags, differential adjustment speeds reflecting cross-market differences in liquidity or transaction costs, or (in some markets) infrequent trading. In addition, futures prices suffer from bid–ask noise. Lastly, futures data have ever-changing maturities, whereas the hedger is interested in the joint distribution of a spot value and a futures price for a single, known time to maturity. The familiar effect of all these errors-in-the-regressor is that the estimated slope coefficient is biased towards zero. A second problem, next to errors in variables, that may plague regression-based hedging strategies is that the joint

² See, for instance, Ederington (1979), Grammatikos and Saunders (1983), Hill and Schneeweis (1982), Stoll and Whaley (1990), Stoll and Whaley (1993, Chapter 4). Kroner and Sultan (1993) improve on the conditional hedge by using a GARCH/ECM model.

distribution of the variables of interest may not be constant over time – that is, unconditional estimates from the past may be very different from conditional, forward-looking parameters. Kroner and Sultan (1993), for example, illustrate how in a one-week delta hedge the use of a bivariate GARCH error-correction model allows one to reduce the variance of the hedged cash flow by about 6% in-sample, and 4.5% out-of-sample, relative to OLS on first differences. A third problem is that, even with error-free data and a constant joint distribution, there always is estimation error because any real-world sample is finite.

Our own work on the optimal implementation of the Johnson–Stein hedge very much relates to earlier results obtained by Kroner and Sultan (1993) and Stoll and Whaley (1993). Like Kroner and Sultan, we compare the out-of-sample performances of various estimation techniques and of price-based hedging rules, and the markets we select for our performance race are currency markets. However, as in Stoll and Whaley, we focus on the impact of errors in variables and the (related) issue of optimal observation frequencies in the regressions.³ Relative to Kroner and Sultan, the innovations in our work are as follows. First, we consider cross hedges, delta hedges, and cross-and-delta hedges rather than just delta hedges. Second, we seek to improve on an unconditional regression not by using GARCH models – at our three-month exposure horizon, GARCH effects are virtually absent – but by using various predictors of the future cross rate or basis that are all implicit in current asset prices or interest rates, spot or forward. We find that, for delta hedges, these price-based hedging rules uniformly beat any regression-based hedge, a result that is very different from Kroner and Sultan. For cross hedges (not studied by Kroner and Sultan), we obtain the same conclusion except for pairs of currencies that are so weakly related that no practitioner would use them as hedges of each other. Lastly, given that one nevertheless uses regression, we find that the choice of the regression technique (OLS versus IV), data frequency, and sample period are, most of the time, far more important than the improvements Kroner and Sultan achieve with a GARCH-ECM model.

Our extensions of Stoll and Whaley's work are in the following directions. First, we use a wider range of regression estimators. For example, we introduce, next to OLS, the Scholes–Williams (SW) instrumental-variable estimator (which takes care of poor synchronization and other lead-lag patterns) and we apply these techniques not just on first-differenced data, but also on percentage-change variables for various sample lengths and frequencies. We find that

³ In a nutshell, choosing a high observation frequency offers the advantage of a larger sample without having to go back far into the past. But the cost of higher-frequency data is that the errors-in-variables bias becomes more acute: the higher the observation frequency, the smaller the signal (the change in true futures price) relative to the noise (caused by bid–ask bounce or imperfect synchronization in the data).

the use of percentages instead of first differences is systematically recommendable, and that the SW estimator beats OLS for cross hedges that, as is invariably the case in practice, involve closely related currencies. Second, we attempt to isolate problems of the errors-in-the-regressor type from problems associated with inevitable estimation noise or changes in the relationship between spot and futures prices. Specifically, we eliminate regressor errors by using currency forward prices – computed from midpoint spot and interest rate data – instead of actual currency-futures prices. Thus, the inferior performance of regression-based hedging strategies relative to price-based hedge ratios cannot be blamed on errors in variables.

The paper is structured as follows. Section 2 briefly reviews the problem and its theoretical solution. In Section 3 we set out the competing price- and regression-based implementations. Section 4 describes the data and presents the results. The conclusions are summarized in Section 5.

2. The problem

In the problem we consider there is one unit of currency i , whose value at time T_1 is uncertain and needs to be hedged. A futures contract is available for a related exchange rate j with an expiry date $T_2 (\geq T_1)$. This general cross-and-delta hedge problem comprises as special cases the perfect hedge ($T_1 = T_2, j = i$, implying $\tilde{f}_{j,T_1,T_2} = \tilde{S}_{i,T_1}$); the cross hedge ($T_1 = T_2, j \neq i$ implying $\tilde{f}_{j,T_1,T_2} = \tilde{S}_{j,T_1}$); and the delta hedge ($T_1 < T_2, j = i$). The size of the futures contract is one unit of the underlying j . Contracts are assumed to be infinitely divisible; that is, one can buy or sell any fraction of the unit contract. Only one type of futures contracts is being used as a hedge.

Denote the number of futures contracts sold by β_t (where t is the current time, when the hedge is set up⁴), the stochastic time- T_1 spot value of asset i by \tilde{S}_{i,T_1} , and the time- T_1 futures rate for asset j and expiry date T_2 by \tilde{f}_{j,T_1,T_2} . Ignoring the (small) time-value effect of marking to market, the hedged cash flow at time T_1 equals $\tilde{S}_{i,T_1} - \beta_t(\tilde{f}_{j,T_1,T_2} - f_{j,t,T_2})$, and the conditional variance-minimizing hedge ratio is

$$\beta_t = \frac{\text{cov}_t(\tilde{S}_{i,T_1}, \tilde{f}_{j,T_1,T_2})}{\text{var}_t(\tilde{f}_{j,T_1,T_2})}, \quad (1)$$

⁴ For convenience, this notation ignores the fact that the optimal β depends on the horizon T_1 and the futures' expiry date T_2 .

where the time subscripts to $\text{var}()$ and $\text{cov}()$ stress the conditional nature of the parameters. The beta in (1) is the population slope coefficient in the linear decomposition of the relation between the future spot and futures rates,

$$\tilde{S}_{i,T_1} = \alpha_t + \beta_t \tilde{f}_{j,T_1,T_2} + \tilde{\varepsilon}_t, \quad (2)$$

where the regressor simplifies to the change in j 's spot rate in the case of a pure cross hedge, or the change in i 's “own” futures price in the case of a pure delta hedge. As the joint distribution of \tilde{S}_{i,T_1} and \tilde{f}_{j,T_1,T_2} is unknown, it has become common practice to estimate β_t from a regression on (suitably differenced) past data. In doing so, the issues are, first: (a) what estimator is to be used, taking into account the statistical properties of the data series; (b) what differencing interval is to be chosen; and (c) whether one should consider simple first differences or percentage changes. A second issue is how to reduce the data problems inherent in futures prices. A last question is whether simple price-based hedge rules may not provide useful alternatives to regression-based estimators. We discuss our procedures in the next section.

3. Regression-based vs price-based hedging rules

Our tests are carried out in currency markets. The main reason is that in this market the problems of poor synchronization, bid–ask noise, and variability in the time to expiry are easily eliminated: we just use, instead of actual futures quotes, theoretical forward prices computed from synchronized spot prices and net convenience yields for the exact maturity needed for the hedging problem at hand.⁵ Thus, we can use virtually noise-free data in the regressions. Ruling out errors in the regressor has the advantage that, in the race between regression-based and price-based hedging rules, the dice are no longer loaded against the former. Thus, if notwithstanding the noise-free data the price-based rules still do better than the regression-based hedging ratios, then we can safely conclude that: (a) the regression-based hedges suffer from more fundamental problems than just noise in the regressor (like substantial estimation errors, or changes in the statistical relationship over time); and (b) when using actual futures data as regression inputs, price-based rules must be even more recommendable.

⁵ To be true, one can compute only theoretical *forward* prices, but these are virtually indistinguishable from theoretical futures prices; see e.g. Cornell and Reinganum (1981). In the case of a stock market hedge, as in Stoll and Whaley (1993), one component of the net cost of carry – the present value of the dividends – is unobservable, so that no noise-free shadow futures prices can be computed. In currency markets, however, the net convenience yields are observable from “swap” forward quotes or can be computed from interbank interest rates.

There are some additional benefits to using computed forward rates. For example, forward rates can be computed for any exchange rate with unrestricted money markets. Thus, the analysis is not confined to currency pairs for which a futures contract is actually traded in the US. This allows us to go beyond pure field tests and set up something of a laboratory experiment. For instance, one can obtain a wide sample of closely related currencies – like the BEF–NLG pair – which can then be compared with currency combinations that are less closely related. Lastly, the availability of an exact theoretical forward rate for a currency hedge allows us to formulate additional, and somewhat more subtle, price-based rules than the one employed by Kroner and Sultan. This is the topic of the next section.

3.1. Price-based forward-looking estimators for currency hedges

In our empirical tests, regression-estimated hedge ratios are competing against strategies that require no statistical analysis of past data but just rely on the most recent price and interest rate data. To understand the logic of these price-based hedge ratios, start from a delta hedge.

In their study of delta hedges, Kroner and Sultan introduce a benchmark rule which consists of setting the sizes of the spot and futures positions equal to each other (that is, the benchmark is $\beta = 1$). This implicitly assumes that, between times t and T_1 , the expected (dollar) changes in the spot rate and the futures price are identical, that is, the basis is not expected to change between times t and T_1 even though the contract's time to maturity will drop from its current level, $(T_2 - t)$, to $(T_2 - T_1)$. To avoid this unattractive assumption, we consider the following alternative predictors of the future basis in a delta hedge:

- in the random walk (RW) model, the forecast of the future basis is the currently observed basis for the same time to maturity, which can be computed from the time- t spot domestic and foreign interest rates for time to maturity $(T_2 - T_1)$;
- alternatively, in the unbiased expectations (UE) model the best possible forecast of the future basis is the “forward” basis for time T_1 against time T_2 , which can be computed using the time- t forward interest rates $T_1 - t$ against $T_2 - t$.

In Appendix A, this approach is formalized and generalized to cross hedges. The corresponding price-based alternative predictors of the cross-rate in a cross hedge are:

- the cross-hedge random walk (RW) model: the best possible forecast of the future cross rate is the currently observed cross rate;
- the cross-hedge unbiased expectations (UE) model: the best possible forecast of the cross rate is the currently observed forward cross rate.

For a cross-and-delta hedge, lastly, the above two pairs can be combined into four different combinations, that is, RW/RW, RW/UE, UE/RW, and UE/UE, where the first entry refers to the predictor used for the cross rate and the second one to the predictor for the basis.

We see three a priori motivations for considering price-based hedge strategies next to regressions. First, a potentially important advantage of simple a priori strategies, like Kroner and Sultan's " $\beta = 1$ " or the closely related rules we just proposed, is that while they may very well be biased, they do avoid estimation errors. In the stock market literature, for instance, Brown and Warner (1980) show that, in event studies that require a market sensitivity or beta, the assumption of unit betas actually does less harm than the estimation errors introduced by theoretically superior regression estimates. Second, regression-based rules use only past information and assume that this past information is sufficient and fully relevant. Price-based hedge ratios, in contrast, are forward-looking and contain only information that the market considers important. For example, when the FRF devalues relative to the DEM, a regression coefficient obtained from first differences will contain lots of old (pre-realignment) data and will therefore adjust to the break in the data only slowly, as more and more post-realignment data enter the sample; in addition, this regression hedge ratio ignores any other information that may be relevant. The most recent spot or forward rate, in contrast, will immediately and fully adjust to any break in the data or any change in the relation between spot and futures prices. A third reason for introducing simple rules is that, among practitioners, (and perhaps because of the two reasons we just gave) regression is far less popular than in the theoretical literature. Thus, our tests evaluate a priori rules that are close to the ones adopted in the Street.

Price-based hedges also have conceptual drawbacks. Specifically, it is not obvious whether either of the assumptions (UE or RW) is actually more appropriate than the assumptions implicit in a regression approach; and it is unlikely that the best possible predictor of, say, the future basis is also the best possible estimator of a conditional delta-hedge regression coefficient. As the choice between regression- and price-based estimators cannot be made on a priori grounds, we chose on empirical grounds. Our regression-based hedge ratios are proposed in the next section.

3.2. Estimation of regression-based hedge ratios

In the forward-looking regression (2), we replace the noisy futures price f_j by the forward price, F_j . In estimating (2) from past data, one typically assumes that β_t is an intertemporal constant and that α_t is, at most, linear in time. Differencing (2) so as to eliminate problems of non-stationarity, and setting $\alpha_t - \alpha_{t-1} = \alpha'_t$, we obtain the regression equation that is standard in this literature:

$$[S_{i,s} - S_{i,s-1}] = \hat{\alpha}'_t + \hat{\beta}'_t [F_{j,s,T_2(s)} - F_{j,s-1,T_2(s-1)}] + e'_s, \quad (3)$$

$$s = t - \text{Nobs}, \dots t - 1.$$

In the regressor of (3), subscript $T_2(s)$ refers to the expiry date of the contract used in the regressions, typically the nearest available one. The regressor still equals the change of the spot rate of currency j , $S_{j,s} - S_{j,s-1}$, in the special case of a cross hedge, and to the change in the exposure-currency's "own" futures price in case of a delta hedge.

Eq. 3 may suffer from heteroscedasticity in the variables if their levels change substantially through time. Following the common view in capital market studies that percentage changes are closer to being identically distributed than dollar price changes, one could also consider

$$\left[\frac{S_{i,s}}{S_{i,s-1}} - 1 \right] = \hat{a}_t + \hat{b}_t \left[\frac{F_{j,s,T_2(s)}}{F_{j,s-1,T_2(s-1)}} - 1 \right] + e''_s, \quad (4)$$

$$s = t - \text{Nobs}, \dots t - 1,$$

and then extract the hedge ratio from the elasticity (b) as follows:

$$\hat{\beta}_t = \hat{b}_t \frac{S_{i,t-1}}{F_{j,t-1,T_2(t-1)}}, \quad (5)$$

where $t-1$ refers to the last day in the estimation sample.

Another problem that may affect either (3) or (4) is imperfect synchronization between regressor and regressand data, especially at high observation frequencies. True, our data should not suffer from any (spurious) lead-lag relationships due to imperfect time matching; still, for intra-European currency pairs the exchange rate mechanism (ERM), or managed floating, may very well introduce (non-spurious) cross-correlations among changes in two exchange rates.⁶ In the presence of cross-correlations, OLS estimators that consider only contemporaneous returns will underestimate the link between the two currencies as soon as the hedging horizon exceeds the observation period adopted in the regression. Accordingly, we also introduce the Scholes and Williams (1977) (SW) instrumental-variable estimator, which is designed to pick up lagged responses between the regressor and the regressand:

$$SW \text{ estimator} = \frac{\widehat{\text{COV}}(\Delta S_{i,s}, IV_{j,s})}{\widehat{\text{COV}}(\Delta F_{j,s}, IV_{j,s})}, \quad (6)$$

⁶ When there is an ERM band or an informal target zone linking two currencies, exchange rate changes relative to the USD must be either perfectly identical (which we know is not the case), or they must follow each other's movements within a relatively short time span – thus creating a lead-lag relation akin to the one caused by poorly synchronized data.

where the SW instrumental variable, $IV_{j,s}$, equals $\Delta F_{j,s-1} + \Delta F_{j,s} + \Delta F_{j,s+1}$.⁷ If the regression variables are percentage changes, the Δ -operator is to be interpreted as a percentage change.

We estimate (3) and (4) using OLS and SW using various sampling frequencies and periods. To streamline the programming of the regressions, we used either all London working days (“daily”), or every fifth working day (“weekly”), or every tenth working day (“biweekly”), or twentieth working day (“monthly”). For daily and weekly sampling, we use two years of data. A two-year interval leaves few observations for regressions with biweekly and (a fortiori) monthly sampling, so for these frequencies we also show results from four-year samples.⁸

4. Data and results

4.1. Data

We select eleven countries that have at least eleven years of daily data (June 1985–December 1996) in the Datastream data base: Belgium, Canada, Denmark, France, Germany, Italy, Japan, the Netherlands, Switzerland, the UK, and the US. Exchange rates are Barclays bank USD quotes against other currencies, except for the BEF/USD rate which is from National Westminster. Forward rates were computed following the procedure outlined in Appendix B.

⁷ See Apte et al. (1994) for a theoretical justification and application of the SW estimator to lead-lag situations other than those caused by thin trading. As in Apte et al. (1994), or Fowler and Rorke (1983), one could extend the lead-lag window to more than one period (one day here), but tests in Sercu and Wu (1999) reveal that there are no important cross-correlations beyond the one-day interval.

⁸ Our use of computed forward prices eliminates errors in the regressor as a source of bias, the focus of this article and also the prime problem discussed in Stoll and Whaley (1993). In contrast, Kroner and Sultan (1993) stress over-differencing of the data and GARCH effects as potential shortcomings in standard regression tests. While their results are positive and interesting, in the case of cross- and cross-and-delta hedges there are practical problems in implementing a GARCH error-correction model. Specifically, while there is little a priori doubt that spot and forward rates for one given currency (as in a delta hedge) are cointegrated, for a cross-hedge or a cross-and-delta hedge the existence of a cointegration relation between non-related currencies is not clear at all. And for EMS pairs, the relation imposed by the exchange rate mechanism is not constant over time, being subject to “trend breaks” (realignments) that are, *ex ante*, difficult to predict. Thus, it is not clear how an ECM for cross hedges should be constructed and estimated. In addition, over a three-month horizon GARCH effects are less important than over a one-week interval. For these reasons, our regression-based hedge ratios are confined to standard estimation techniques.

From the total menu of 45 possible pairs that could enter into a cross-hedge problem, we select three groups of country pairs, in a way that should provide a sufficient variability in the degree of relatedness between the two members of the pair.

Strongly related currencies: The first group contains intimately related currency pairs that US-based traders would surely consider to be excellent candidates in a cross hedge: NLG and BEF (where for most of the sample period an intra-Benelux agreement imposed a 1%-band around the ERM central rate), DEM and NLG (which the Nederlandse Bank unilaterally kept within a narrow band for most of the sample period), and lastly BEF and DEM (linked indirectly through the above arrangements, and directly by unilateral intervention by the Nationale Bank van België).

Ordinary (quasi-)ERM pairs: The second group contains a straight ERM pair (DKK and FRF), two combinations between an ERM currency and the CHF (which, until mid-1997, was widely viewed as linked to the DEM, even though Switzerland's central bank denies that it actually intervenes in exchange markets), and the ITL-GBP pair. These four pairs still show substantial common characteristics, although less so than the first group.

Weakly- or un-related currencies: To verify whether the differences between the two above groups can be extrapolated to even less related currency pairs, we also consider GBP-CAD and ITL-JPY. In fact, the only commonalities between the two members of each pair probably is the USD-component in the exchange rates, and it extremely doubtful whether, in reality, a practitioner would ever hedge a GBP exposure using CAD, or an ITL exposure using JPY.

4.2. Test procedure

We set aside the first four years of data for the initial estimation of the regression coefficients. Thus, at the beginning of the 49th month of data we determine the hedge ratio, using either the beginning-of-the-month prices (for the price-based rules) or two to four years of daily, weekly, biweekly, or monthly data (for the regression-based estimators). Let the competing estimation rules be indicated by subscripts $h = 1, \dots, H$. For each of these betas $\hat{\beta}_t^h$, the cash flow hedged in month t is then computed as $Z_t[\tilde{S}_{i,T_1(t)} - \hat{\beta}_t^h(\tilde{F}_{j,T_1(t),T_2(t)} - F_{j,t,T_2(t)})]$ where Z_t is the contract size (which we set equal to either $1/S_{i,t}$ or unity). This cash flow is usually non-stationary, and so is its conditionally stochastic component, $Z_t[\tilde{S}_{i,T_1(t)} - \hat{\beta}_t^h \tilde{F}_{j,T_1(t),T_2(t)}]$. To obtain a better-behaved variable, we follow standard procedure and subtract the initial spot rate; that is, we study the variable $Z_t\{[\tilde{S}_{i,T_1(t)} - S_{i,t}] - \hat{\beta}_t^h[\tilde{F}_{j,T_1(t),T_2(t)} - F_{j,t,T_2(t)}]\}$. The entire procedure is repeated for every subsequent month, each time resetting the price-based hedge ratios or re-estimating the

regression coefficients. For each time series of hedge ratios $\{\hat{\beta}_t^h, t = 50, \dots, N\}$ the N monthly hedge errors are then summarized by their mean square (MS):⁹

$$MS_h = \sum_{t=49}^N \frac{\{Z_t[(\tilde{S}_{i,T_1(t)} - S_{i,t}) - \hat{\beta}_t^h(\tilde{F}_{j,T_1(t),T_2(t)} - F_{j,t,T_2(t)})]\}^2}{N - 49} \tag{7}$$

Lastly, to facilitate cross-currency comparison of the hedging methods, each MS is rescaled by dividing it by the MS of the RW (or RW/RW) hedge.

Regarding the size of the contract, we only report results for $Z_t = 1/S_{i,t}$, that is, where each month the number of foreign currency units that is being hedged corresponds to a book value of USD 1. Thus, we report results for a time series of dimensionless percentage numbers, namely $[(\tilde{S}_{i,T_1(t)} - S_{i,t}) - \hat{\beta}_t^h(\tilde{F}_{j,T_1(t),T_2(t)} - F_{j,t,T_2(t)})]/S_{i,t}$. In contrast, the standard procedure sets Z_t equal to unity, and works with a time series of dollar amounts per unit of foreign currency. Relative to this standard approach, percentages offer the advantages that (a) the division by the initial level eliminates an obvious source of heteroscedasticity and (b) the resulting measures of volatility are more comparable across currencies. In fact, percentage changes are the standard transform in empirical studies of speculative markets. Results for dollar amounts are available on request. The assumption about Z_t has, of course, a marked effect on the absolute size of the MSs across exchange rates, but hardly affects the ratios of different MSs for one given exchange rates and in no way changes any conclusion.

4.3. The performance of price-based hedges

To set the benchmark for the regression-based hedges, Tables 1 and 2 the results from price-based strategies. Table 1 shows the root¹⁰ MS cash flow of the exposed currency, first without hedging and then after applying the RW hedge in, respectively, a cross-and-delta problem, a pure cross hedge, and a pure delta hedge. (As we shall see, the other price-based rules have a very

⁹ The rankings are not affected when the mean is subtracted, i.e., when the variance is computed rather than the MS error. We prefer the latter because the mean is insignificantly different from zero and not known, *ex ante*, to the trader. Nor are the ranking affected if one relies on mean absolute deviation rather than on MSs to evaluated the competing hedging rules.

¹⁰ In Table 1 we use the *root* mean square for the purpose of showing risk as an absolute magnitude, because the root mean square percentage change is almost indistinguishable from the volatility of log changes (the standard measure of risk in option pricing). In all subsequent tables, in contrast, we use the mean squares themselves, as standard in the hedging literature, and we deal with the divergent magnitudes by dividing this MS by the MS of the RW-based rule.

Table 1
Root MS cash flow, in percent per quarter, before hedging and after applying the RW-based hedge^a

Not hedged		RW cross-and-delta hedge			RW cross hedge			RW/RW delta hedge		
Currencies <i>i</i> and <i>j</i>	RMS (%/quarter)	Currencies <i>i</i> and <i>j</i>	RMS (%/quarter)	% Risk reduction	Currencies <i>i</i> and <i>j</i>	RMS (%/quarter)	% Risk reduction	Curren- cy (<i>t</i> = <i>t</i>)	RMS (%/quarter)	% Risk reduction
DEM	n.a.	DEM	0.26795	96	NLG	DEM	0.24010	DEM	0.17021	97
NLG	n.a.	NLG	1.07683	82	BEF	NLG	1.08522	NLG	0.26349	99
BEF	n.a.	BEF	1.26694	79	DEM	BEF	1.09797	BEF	0.16317	97
DKK	n.a.	DKK	1.02530	82	FRF	DKK	0.96754	DKK	0.3207	94
CHF	n.a.	CHF	2.46840	59	DKK	CHF	2.41083	CHF	0.4377	93
FRF	n.a.	FRF	2.31984	64	CHF	FRF	2.29890	FRF	0.20071	97
GBP	n.a.	GBP	3.64060	43	ITL	GBP	3.60859	GBP	0.33057	95
CAD	n.a.	CAD	5.77030	5	GBP	CAD	5.78942	CAD	0.19197	97
ITL	n.a.	JPY	8.13144	-31	JPY	ITL	8.16641	ITL	0.14255	98

^a At the beginning of each month *t* from June 1989 through December 1996, a three-month exposure in currency *i* is hedged using β futures contracts of hedge currency *j*. In the panels “cross-and-delta hedge” and “delta hedge”, the futures contract is a six-month contract while in the column “cross hedge” it is a three-month contract. The scaled hedged cashflow per period is

$$\frac{[S_{i,T_1} - S_{i,t}] - \beta_i^h [f_{j,T_1(t),T_2(t)} - f_{j,t,T_2(t)}]}{S_{i,t}}$$

where *S* is a spot rate, *f* is an error-free substitute for the the futures price (a forward price computed from midpoint spot and interest rates), *t* is the date of initiation of the hedge, *T*₁ is the hedge horizon (*t* plus three months), and *T*₂ is the expiry date of the hedge (*t* plus three or six months). The hedge ratio β_i^h is set on the basis of the Random Walk model (RW), that is, assuming that cross exchange rates or interest rates will not change over the next three months. The columns “RMS” show the root mean square of each hedged cashflow, in percent per quarter. The percentage risk reduction is computed as

$$\left[\frac{RMS_{\text{hedged}} - RMS_{\text{unhedged}}}{RMS_{\text{unhedged}}} \right] \times 100.$$

Table 2
MS comparison among price-based hedges: MS cash flow of price-based hedges, scaled by the MS cash flow of the RW hedge^a

Cross-and-delta hedge		MS ratio for rule		Cross hedge		Delta hedge	
Currencies <i>i</i> and <i>j</i>		UE/RW		RW/UE		Currencies <i>i</i> and <i>j</i>	
		UE/UE		UE/UE		MS ratio for UE	
DEM	NLG	1.006	0.990	DEM	NLG	DEM	1.010
NLG	BEF	0.996	1.000	NLG	BEF	NLG	0.994
BEF	DEM	1.000	0.998	BEF	DEM	BEF	1.000
DKK	FRF	1.008	0.990	DKK	FRF	DKK	0.992
CHF	DKK	0.986	0.998	CHF	DKK	CHF	0.960
FRF	CHF	0.996	1.002	FRF	CHF	FRF	1.002
GBP	ITL	1.004	1.000	GBP	ITL	GBP	1.024
CAD	GBP	1.002	1.000	CAD	GBP	CAD	1.028
ITL	JPY	1.022	0.998	ITL	PY	ITL	0.998

^a At the beginning of each month *t* from June 1989 through December 1996, a three-month exposure in currency *i* is hedged using β futures contracts of hedge currency *j*. In the panels “cross-and-delta hedge” and “delta hedge”, the futures contract is a six-month contract while in the column “cross hedge” it is a three-month contract. The scaled hedged cashflow per period is

$$\frac{[S_{t,T_1} - S_{t,t}] - \beta_t^i [f_{j,T_1(t),T_2(t)} - f_{j,tT_2(t)}]}{S_{t,t}}$$

where *S* is a spot rate, *f* is an error-free substitute for the the futures price (a forward price computed from midpoint spot and interest rates), *t* is the date of initiation of the hedge, *T*₁ is the hedge horizon (*t* plus three months), and *T*₂ is the expiry date of the hedge (*t* plus three or six months). The competing hedge ratios are:

- RW (random walk): β_t^i is set assuming that cross exchange rates or interest rates will not change over the next three months.
- UE (unbiased expectations): β_t^i is set assuming that future cross exchange rates or three-month interest rates are well predicted by the corresponding forward rates
- RW/UE, UE/RW, etc.: the first entry refers to the hedging rule used for the exchange rate, the second one to the hedging rule used for the interest rates.

The columns “MS ratio” show the mean square of the hedged cash flow divided by the MS from the RW (or RW/RW) hedging rule. For absolute magnitudes of the latter, see Table 1.

Table 3
MS cash flow from regression-based hedges relative to RW result^a

<i>i</i>	<i>j</i>	Using two years of data						Using four years of data									
		Day	Day	Day	Week	Week	Month	2 week	2 week	2 week	Month	Month	Month	Month	Month	Month	
		SWA	SW%	OLS%	OLS	OLS%	OLS%	OLS	OLS%	OLS	OLS%	OLS	OLS%	OLS	OLS%	OLS	OLS%
Panel A: Cross- and delta-hedges																	
DEM	NLG	1.186	1.184	2.477	2.430	1.414	1.399	1.120	1.084	1.19	1.208	1.107	1.084	1.080	1.038		
NLG	BEF	1.077	1.010	1.390	1.313	1.156	1.084	1.125	1.056	1.146	1.072	1.124	1.034	1.084	1.002		
BEF	DEM	1.061	1.069	1.156	1.190	1.090	1.092	1.058	1.049	1.067	1.058	1.042	1.034	1.036	1.034		
DKK	FRF	0.882	0.870	0.992	0.972	0.901	0.889	0.926	0.914	0.916	0.910	0.947	0.931	0.968	0.956		
CHF	DKK	0.889	0.889	0.922	0.914	0.920	0.901	0.908	0.900	0.907	0.888	0.933	0.933	0.901	0.870		
FRF	CHF	1.040	1.036	1.153	1.153	1.059	1.044	0.994	0.989	1.003	0.998	1.002	0.986	1.044	1.030		
GBP	ITL	0.939	0.941	0.933	0.937	0.931	0.929	0.992	1.000	0.936	0.934	1.047	1.014	1.042	1.002		
CAD	GBP	1.047	1.038	1.012	1.000	1.094	1.090	1.361	1.358	1.134	1.127	1.061	1.049	1.113	1.090		
ITL	JPY	0.645	0.642	0.646	0.643	0.659	0.656	0.654	0.651	0.643	0.639	0.654	0.654	0.664	0.663		
Panel B: Cross-hedges																	
DEM	NLG	1.149	1.141	2.465	2.500	1.416	1.409	1.063	1.034	1.131	1.154	0.986	0.992	1.024	0.996		
NLG	BEF	1.053	1.000	1.336	1.275	1.126	1.067	1.120	1.064	1.111	1.054	1.080	1.020	1.040	0.986		
BEF	DEM	1.075	1.082	1.175	1.221	1.086	1.086	1.055	1.047	1.068	1.061	1.028	1.022	1.026	1.020		
DKK	FRF	0.903	0.897	0.982	0.964	0.872	0.869	0.867	0.874	0.896	0.903	0.960	0.960	0.927	0.937		
CHF	DKK	0.884	0.887	0.920	0.916	0.918	0.901	0.906	0.900	0.900	0.890	0.941	0.941	0.889	0.867		
FRF	CHF	1.059	1.049	1.171	1.164	1.080	1.059	0.995	0.985	1.018	1.007	1.020	0.998	1.061	1.038		
GBP	ITL	0.935	0.939	0.925	0.931	0.925	0.925	0.984	1.000	0.933	0.933	1.026	0.998	1.020	0.988		
CAD	GBP	1.034	1.024	1.008	0.996	1.075	1.071	1.315	1.313	1.121	1.113	1.059	1.047	1.103	1.084		
ITL	JPY	0.640	0.637	0.642	0.637	0.654	0.651	0.651	0.648	0.637	0.633	0.648	0.648	0.656	0.654		

Panel C: Delta-hedges

$i=j$	1.173	1.014	1.175	1.012	1.201	1.026	1.297	1.072	1.281	1.077	1.357	1.036	1.464	1.119
DEM	1.111	1.032	1.117	1.032	1.145	1.073	1.190	1.087	1.199	1.100	1.179	1.082	1.151	1.059
NLG	1.332	1.107	1.320	1.094	1.350	1.115	1.376	1.109	1.366	1.113	1.488	1.080	1.605	1.173
BEF	1.032	1.010	1.038	1.024	1.055	1.036	1.067	1.072	1.062	1.056	1.008	1.024	0.988	0.998
DKK	0.861	0.920	0.925	0.964	0.933	0.988	1.013	1.074	0.949	1.011	0.922	0.922	0.943	1.020
CHF	1.175	1.053	1.158	1.051	1.162	1.042	1.151	0.995	1.147	1.033	1.252	1.047	1.306	1.077
FRF	0.986	0.882	1.040	0.925	0.941	0.878	0.925	0.925	1.020	0.918	1.098	0.947	1.105	0.956
GBP	1.132	1.018	1.134	1.040	1.166	1.057	1.204	1.032	1.337	1.183	1.631	1.383	1.721	1.421
CAD	1.182	1.044	1.177	1.057	1.212	1.055	1.395	1.180	1.250	1.085	1.277	1.069	1.383	1.134
ITL														

^a At the beginning of each month t from June 1989 through December 1996, a three-month exposure in currency i is hedged using futures contracts of hedge currency j . In the panels “cross-and-delta hedge” and “delta hedge”, the futures contract is a six-month contract while in the column “cross hedge”, it is a three-month contract. The scaled hedged cashflow per period is

$$\frac{[S_{i,T_1} - S_{i,t}] - \beta [f_{j,T_1(0),T_2(0)} - f_{j,t,T_2(0)}]}{S_{i,t}}$$

where S is a spot rate, f is an error-free substitute for the the futures price (a forward price computed from midpoint spot and interest rates), t is the date of initiation of the hedge, T_1 is the hedge horizon (t plus three months), and T_2 is the expiry date of the hedge (t plus three or six months). The base case for comparison is to set the hedge ratio assuming that cross exchange rates and/or interest rates will not change over the next three months (Table 1), and all MSs for regression-based hedges are divided by the MS of this base case. In the competing regression-based hedges, is set using the following techniques:

- OLSA: OLS regressions on first differences (daily to monthly).
- OLS%: OLS regressions on percentage returns (daily to monthly), with the slope rescaled into a hedge ratio using time- t rates, see Eq. (5).
- SWA: Scholes-Williams regressions on first differences (daily).
- SW%: Scholes-Williams regressions on percentage returns (daily), with the slope rescaled into a hedge ratio using time- t rates, see Eq. (5).

similar performance.) First consider the delta hedge, as studied by Kroner and Sultan. In all cases, the price-based delta hedge reduces the volatility by 93% (DKK) to almost 98% (JPY). For cross hedges and cross-and-delta hedges, in contrast, the performance of the hedge predictably depends much more on the degree of relatedness of the two pairs. For ERM pairs, applying the random-walk rule of thumb reduces the risk by some 82% (hedging BEF using NLG) to 96% (NLG by DEM); for non-ERM European pairs the risk-reduction ranges from 60–65% (the cases involving the CHF) to a lowish 33% (hedging ITL using GBP). For unrelated pairs, lastly, price-based hedging achieves virtually no risk-reduction (CAD–GBP), or may actually backfire rather badly (ITL–JPY); recall, however, that the last two combinations are a priori not realistic for hedging purposes. We also see that the results for cross-and-delta hedges are quite close to those of pure cross hedges; that is, cross-rate volatility is the dominant source of basis risk in a cross-and-delta risk, and the delta-component is rather marginal.

Table 2 illustrates how the other price-based hedging rules perform relative to the no-change rule. In Table 2, as in Table 3 discussed below, all mean squares are rescaled by the MS cash flow of the RW or RW/RW rule. All ratios in Table 2 turn out to be extremely close to unity. Thus, even though it is widely accepted that the RW model beats the UE model as an exchange rate forecaster (Froot and Thaler, 1990), for current purposes the two models are indistinguishable, and our choice of the no-change rule as the benchmark in Tables 2 or 3 is not crucial. The more interesting question, then, is how the price-based hedging rules fare relative to the regression-based strategies.

Table 3 shows the MS ratios for regression-based hedges. For ease of comparison, the results for OLS regressions using two years of data (with varying observation frequency) are presented in the central part of the table. To the left of the MS ratios for daily OLS regressions, we present the ratios for daily SW regressions; and to the right we add the numbers for biweekly or monthly OLS regressions obtained with four rather than with two years of data. Comparing price- and regression-based results, we note that unlike in Kroner and Sultan (1993) MS ratios in excess of unity are by no means the exception. In fact, regressions do systematically poorly for delta hedges, as well as for cross hedges or cross-and-delta hedges that involve strongly related currencies. The only case where a regression markedly beats a price-based hedge – JPY exposure being covered in the ITL market – is a combination that nobody would actually have chosen in practice. Given that, in this test, one cannot invoke regressor errors as an explanation of the less-than-impressive performance of the statistics-based hedge ratios, we conclude that the regressions must suffer from high estimation variance and/or from some form of misspecification. We return to this issue below.

4.4. Comparing different regression-based strategies

Some additional interesting patterns emerge when one has a closer look at the regressions. First, in this study the differences of performance are quite large relative to what Kroner and Sultan observe. For example, within the class of regression estimators the choice of a sample (period length and observation frequency) or of an estimator (OLS vs SW) matters far more than the choice of OLS vs GARCH-ECM, which in Kroner and Sultan (1993) improves the MS by just 4.5%. The differences between regression- and non-regression-based strategies for delta hedges are much larger than in Kroner and Sultan, too; and for cross- or cross-and-delta hedges, which Kroner and Sultan do not examine, the performance gaps are even more pronounced. A second general observation is that, for a given sample and estimation technique, the results from regressions using percentage-change data are virtually always better than the ones from regressions between first differences – even in two-year samples, where the variability in the level of the exchange rates is clearly lower than in four-year samples. This finding confirms the standard view that, for asset prices, percentage changes have better statistical properties than dollar price changes. Witt et al. (1987) find that, for commodity prices, first differences do a better job.

Closer inspection of the regression MS ratios provides some clues why the OLS-based strategies do poorly. We shall argue that, for cross- or cross-and-delta hedges between closely related currencies the source of the problem is cross-correlations, while for delta hedges the reason is more likely to be a changing relationship over time.

To substantiate the first claim, we note that the regression-based cross hedges that use two years of data have the following characteristics (apart from doing worse than price-based hedges): (a) the low-frequency regressions do substantially better than high-frequency regressions; and (b) for daily observations, SW resoundingly beats OLS. These findings imply that, at high frequencies, the ERM does induce lead–lag patterns or cross correlations. The SW estimator performs better because it is designed to pick up precisely these cross-correlations. If short-term cross-correlations are indeed the main problem, they should also be picked up by increasing the observation interval. In fact, we see that even though sample sizes become smaller and smaller, weekly OLS does better than daily, and biweekly better than weekly OLS and even SW.¹¹ Thus, for cross- or cross-and-delta hedges involving

¹¹ However, when going from biweekly to monthly data, the advantage of picking up more lead/lag relations appears to be more than compensated by the concomitant loss of degrees of freedom. When, accordingly, the sample period is increased to four years, monthly sampling comes out as the winner; in fact, for closely related currencies the results for four years of monthly data become close to the ones from the price-based rules.

highly related currencies, lead–lag patterns seem to be the prime source of problems in the high-frequency OLS regressions with two years of data. To confirm this picture, we note that none of these patterns is present in the group of unrelated currencies – the combinations that no real-world treasurer would actually select for a cross- or cross-and-delta hedge. In this third group, there is no clear association between MS ratio and sample period or frequency, and SW does not improve on OLS. In the absence of an obvious mis-specification problem, the regression does about as well as the price-based rule (CAD–GBP), or substantially better (ITL–JPY)¹². The diagnosis for group 2, finally, is somewhere in between: there is some evidence of cross-correlations (as shown by the superiority of daily SW, or two years of biweekly data or four years of monthly data relative to daily – all relative to daily or weekly OLS); but the price-based rules do not systematically outperform the regression-based hedge ratios, and increasing the sample size does not help.

In contrast, for pure delta hedges (Panel C) there is no a priori reason to expect EMS and quasi-EMS currencies to be very different from others; nor do we see any such difference in the figures. There is no evidence of lead-lag relationships either; SW is typically quite close to OLS, and the differences between the MS cash flows of these two go either way, without any clear pattern. As we found for cross hedges that involve unrelated currencies, for delta hedges a sample of recent high-frequency data does better than low-frequency data; and increasing the sample period by going back four years instead of two actually worsens the results. This suggests that the main problem that plagues delta-regressions seems to be a changing relationship between the regression variables. Kroner and Sultan's finding that, at the one-week horizon, GARCH-ECM does better than OLS points in the same direction.

5. Conclusions

When hedging an asset using a futures contract that has the wrong expiry data, or the wrong underlying asset, or both, the variance-minimizing hedge ratio depends on unobservable conditional parameters, which have to be estimated. If unconditional regression analysis of past data is used, the issues are: (a) what estimator is to be used, taking into account the statistical properties of the data series; (b) what differencing interval is to be chosen; and (c) whether one should consider first differences or percentage changes. A more radical

¹² In fairness, recall that in this particular case the application of the naive rule actually increased the risk. It can easily be calculated that the regressions reduce the total variability by about 1/6.

question is whether simple price-based rules provide useful alternatives, or complements, to regression-based estimators.

In this paper we find that, regardless of observation frequency and estimation technique, unconditional backward-looking regressions are often poor proxies for the ideal regression, even to the extent that regression-based hedges are usually beaten by simple price-based rules. For delta hedges, this effect is rather pervasive, while for cross hedges and cross-and-delta hedges the superiority of the price-based hedging rule is especially clear among closely related currencies. As our data are free of most measurement errors, this relatively poor performance of regression-based hedges is unlikely to be due to just errors in data. For cross hedges involving two European currencies, the poor performance of high-frequency OLS estimates can be traced to EMS-induced leads and lags among exchange rate changes, while for delta hedges the dominant source of estimation problems seems to be a time-varying relationship between the regression variables. Lastly, we find that regressions that use percentage returns do better than estimates based on dollar price changes.

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Appendix A. The forward-looking hedging rules

To obtain a simple forward-looking estimator of t , we consider two elementary no-arbitrage conditions. First, forward rates satisfy Interest Rate Parity,

$$\tilde{F}_{j,T_1,T_2} = \tilde{S}_{j,T_1} \frac{1 + \tilde{r}_{T_1,T_2}}{1 + \tilde{r}_{T_1,T_2}^j}, \quad (\text{A.1})$$

where r_{T_1,T_2} is the effective rate of return, without any annualization, on a risk-free investment between times T_1 and T_2 in the domestic currency (USD), and r_{T_1,T_2}^j is the effective return on the currency- j risk-free investment. Rearranging (A.1), we obtain the following relation between the spot value of the hedge currency and its futures price:

$$\tilde{S}_{j,T_1} = \frac{1 + \tilde{r}_{T_1,T_2}^j}{1 + \tilde{r}_{T_1,T_2}} \tilde{F}_{j,T_1,T_2}. \quad (\text{A.2})$$

Note that the futures price on the right-hand side is the regressor in (2). The spot rate on the left-hand side of (A.2) is not yet the regressand in (2), except in the case of a pure delta hedge ($i=j$). We can, however, make a link with the regressand by invoking a second arbitrage relationship, triangular arbitrage:

$$\tilde{S}_{i,T} = \tilde{S}_{i,T}^j \tilde{S}_{j,T}, \quad (\text{A.3})$$

where $\tilde{S}_{i,T}^j$ is the cross-rate (the value of the exposure currency, i , in units of the hedge currency, j). Combining (A.2) and (A.3), we obtain the following no-arbitrage condition:

$$\tilde{S}_{j,T_1} = \left[\tilde{S}_{i,T_1}^j \frac{1 + \tilde{r}_{T_1,T_2}^j}{1 + \tilde{r}_{T_1,T_2}} \right] \tilde{F}_{j,T_1,T_2}. \quad (\text{A.4})$$

Both the regressand and regressor of (2) now appear in (A.4). We see that if the time- T_1 cross rate and the interest rates were known, then there would be no need to estimate β_t . In fact, the exposure would be a priori equal to

$$\beta_t = S_{j,T_1}^j \frac{1 + r_{T_1,T_2}^j}{1 + r_{T_1,T_2}} \quad (\text{certainty model}). \quad (\text{A.5})$$

In practice, the future cross-rate and interest rates are, of course, unknown, but we can experiment with simple predictors. For example, under the unbiased expectations (UE) hypothesis we have $E_t(\tilde{S}_{i,T_1}^j) = F_{i,t,T_1}^j$, where F_{i,t,T_1}^j is the forward cross rate. Alternatively, if spot rates are random walks (RW), then $E_t(\tilde{S}_{i,T_1}^j)$ equals $S_{i,t}^j$, the current cross rate. Thus, our alternative price-based estimators for the future spot rate in (A.5) are:

$$\text{UE} : \hat{S}_{i,T_1}^j = F_{i,t,T_1}^j, \quad (\text{A.6})$$

and

$$\text{RW} : \hat{S}_{i,T_1}^j = S_{i,t}^j. \quad (\text{A.7})$$

This already provides two price-based estimators for β_t in a cross-hedge problem (where $T_1 = T_2$, that is, where no future interest rates need to be predicted). Analogously, as alternative predictors for the future interest rates we use either the current relative return ratio for the same time to maturity ($T_2 - T_1$) – the no-change or random-walk (RW) forecast:

$$\text{RW} : \frac{1 + \hat{r}_{T_1,T_2}^j}{1 + \hat{r}_{T_1,T_2}} = \frac{1 + r_{t,t+T_2-T_1}^j}{1 + r_{t,t+T_2-T_1}}, \quad (\text{A.8})$$

or the current forward interest rates – the unbiased-expectations (UE) forecast:

$$\text{UE : } \frac{1 + \hat{r}_{T_1, T_2}^j}{1 + \hat{r}_{T_1, T_2}} = \frac{(1 + r_{t, T_2}^j)/(1 + r_{t, T_1}^j)}{(1 + r_{t, T_2})/(1 + r_{t, T_1})}. \quad (\text{A.9})$$

Expressions (A.8) and (A.9) provide our alternative price-based estimators for the exposure in a delta hedge (where $i=j$, that is, where no future cross rate needs to be predicted). Lastly, for a cross-and-delta hedge we use the four combinations of the random-walk (RW) and unbiased expectations (UE) estimators for the cross rate and the premium.

Appendix B. Computation of the forward prices

To estimate the forward-looking regression $\tilde{S}_{i, T_1} = \alpha_t + \beta_t \tilde{F}_{j, T_1, T_2} + \tilde{\varepsilon}_{t, T_1}$ from past data, we first construct a data series that is clean from errors-in-the-regressor. We consider a hedging horizon, $T_1 - t$, of three months, and we specify that the remaining life of the hedge, $T \equiv T_2 - T_1$, is equal to zero (for a cross hedge) or one quarter (for a delta or cross-and-delta hedge).¹³ Thus, for every date we can compute forward prices with a constant time to maturity of three or six months. This eliminates the change in the life of futures prices as one source of errors-in-variables bias in the regression.¹⁴ In addition, if swap forward quotes are used or if forward rates are computed from spot exchange rates and interest rates or swap rates, then synchronization of the observations is no longer a problem either. Lastly, if midpoint data are used, bid-ask noise is avoided as well. In practice, we have chosen to compute forward rates from interest rates rather than from three- and six-month swap rates, for the following reason. In our story, the hedge is liquidated on the expiry date, T_1 . For this reason we want T_1 to be a working day, a condition that is not always met for the expiry day of standard 90- or 180-day market quotes.¹⁵ Thus, starting

¹³ The use of a three-month horizon has the drawback that there is overlap in the month-by-month hedging errors, but is dictated by data availability. Datastream provides one-, three-, and six-month interest rates, which allows us to analyze a problem of hedging a three-month exposure using a six-month hedge but (because of the absence of two-month Euro-rates) not the problem of hedging a one-month exposure using a two-month hedge. Other data series consulted by us provided much shorter time series and were hard to splice into the Datastream exchange rate files.

¹⁴ One conceivable alternative is to use, from a time series of past futures prices, only the data points that correspond to a three-month remaining life. However, this would dictate the use of very old data if a reasonable sample size is to be obtained. In addition, the data would still suffer from bid-ask noise and synchronization errors.

¹⁵ The delivery day is, of course, always a working day, but this is not always true for the expiration day. For instance, a 90-day contract taken out on February 25, 1997 (a Tuesday) expires on April 25, 1997 (a Sunday). The delivery day would then be April 27 (a Tuesday), but on Sunday April 25 itself we cannot trade.

from every working day t , we first go to the date three months (“90 days”) later; and if this tentative T_1 -date is not a working day, we define T_1 as the first subsequent working day. T_2 is defined similarly. Delivery then takes place on the second working day after this date T_2 , except for the CAD where a one-working-day rule applies (see Grabbe (1996) for details on the time conventions).

To compute the t -to- T_1 forward exchange rate, we next need to consider the replicating deposits (or loans) made at time t . Such a deposit earns interest from the second working day following day t and until the calendar day before the delivery date. We therefore compute the number of interest-earning days between these dates as a fraction of a year, either from the number of calendar days and a 365-day year (the interbank convention for the GBP and the BEF), or using the 30-days-per-month, 360-days-per-year rule applicable for other currencies. We compute the return on the deposit or loan by multiplying the time to maturity, $(T_1 - t)$, by the three-month interest rate; that is, following interbank practice, we ignore the fact that $(T_1 - t)$ may be one or two days off the three-month mark. Our three-month forward rate then follows. For the six-month rate the procedure is analogous, except that we start from date T_1 rather than t . As mentioned before, we use midpoint rates so as to eliminate bid-ask noise.

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