If the short-term rate $r(t)$ is taken to be unobservable, there are four coefficients to be estimated: κ, ϕ_0, ϕ_1 and ϕ_2. From these estimated coefficients we can derive the implied parameters,

$$\text{implied short} - \text{term rate:} \quad r(t) = \kappa\phi_0(t), \tag{9}$$

$$\text{yield on a bond with } T \to \infty: \quad R_L = \kappa\phi_1, \tag{10}$$

$$\text{implied variance of } dr: \quad \sigma^2 = 4\kappa^3\phi_2, \tag{11}$$

$$\begin{aligned}\text{risk} - \text{adjusted drift rate of } r(t): \quad \mu &\equiv \kappa(m-r) - \lambda\sigma \\ &= (\phi_1 + 2\phi_2)\kappa^2 - \kappa r. \end{aligned} \tag{12}$$

In contrast, Cox et al. (1985b) adopt a specific general-equilibrium approach that allows them to derive both the interest rate dynamics and the corresponding price of risk:

$$\mathrm{d}r = \kappa(m-r)\mathrm{d}t + \sigma\sqrt{r}\,\mathrm{d}z, \tag{13}$$

$$\lambda(r,t) = \frac{q}{\sigma}\sqrt{r(t)}, \tag{14}$$

where q is a constant. As a result, the general differential Eq. (2) can be specified as

$$\frac{\partial P}{\partial t} + \frac{\partial P}{\partial r}[\kappa(m-r) - qr(t)] + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}\sigma^2 r - r(t)P = 0. \tag{15}$$

With the boundary condition $P_T(r,T) = 1$ for a maturing discount bond, the solution to Eq. (15) takes the following specific form:

$$P_T(r,t) = \left\{\frac{\theta_1 e^{\theta_2(T-t)}}{\theta_2[e^{\theta_1(T-T)} - 1] + \theta_1}\right\}^{\theta_3} \exp\left\{\frac{-r[e^{\theta_1(T-t)} - 1]}{\theta_2[e^{\theta_1(T-t)} - 1] + \theta_1}\right\}, \tag{16}$$

where

$$\theta_1 = \sqrt{(\kappa+q)^2 + 2\sigma^2}, \tag{17}$$

$$\theta_2 = (\kappa + q + \theta_1)/2, \tag{18}$$

$$\theta_3 = 2\kappa m/\sigma^2. \tag{19}$$

Also in this model there are four coefficients to be estimated: $r(t)$, θ_1, θ_2 and θ_3. From these estimated coefficients we can derive the implied parameters,

$$\text{yield on a bond with } T \to \infty: \quad R_L = \theta_3(\theta_1 - \theta_2), \tag{20}$$

$$\text{implied variance of } \mathrm{d}r: \quad \sigma^2 r(t) = 2\theta_2(\theta_1 - \theta_2)r(t), \tag{21}$$

$$\begin{aligned}\text{risk} - \text{adjusted drift rate of } r(t): \quad \mu &\equiv \kappa(m-r) - qr(t) \\ &= \theta_3\sigma^2/2 - (2\theta_2 - \theta_1)r(t). \end{aligned} \tag{22}$$

The cubic spline model [4], finally, is a purely descriptive model without economic foundations. The term structure function consists of a concatenation of a number of third-degree polynomials, spliced together at n "knot points", s_i, $i = 1, \ldots, n$, in a way that ensures continuity in the levels as well as the first and second derivatives:

$$\begin{aligned} P_T(r,t) = {} & 1 + a_t(T-t) + b_t(T-t)^2 + c_t(T-t)^3 \\ & + \sum_{i=1}^{n} \mathrm{d}_{t,i}[\mathrm{Max}\{T-(t+s_i),0\}]^3. \end{aligned} \tag{23}$$

Usually one selects two knot points — in this study, $s_1 = 2$ years and $s_2 = 4$ years — which implies there are five free parameters in the spline model. As will be illustrated below, in the daily cross-sectional estimations the five-parameter spline tends to produce a better fit than the Vasicek or CIR models. This better fit may stem from two sources: first, this spline has one more free parameter, and second, it imposes less restrictions on the shape of the discount function than the other two models. To be able to sort out the relative importance of each explanation, we