## Appendix R: Technical notes for Footnote 3 (for referee's reference only)

Let $r_{\mathrm{p}}$ be the premium imposed in both good and bad states. We analyze the three scenarios in the new setting, respectively (sticking to the same numbering of equations in the paper).

## Scenario I: The investment decision with bank debt only

Case 1: The manager on behalf of shareholders would like the firm to undertake a project with a non-negative return on equity (under risk neutrality),

$$
\begin{equation*}
N P V_{\text {equity }}=q\left[A+I\left(r_{H}-r\right)\right]+(1-q) \operatorname{Max}\left\{A+I\left(r_{L}-r\right), 0\right\}-A \geq 0 . \tag{2}
\end{equation*}
$$

Here the original rent extraction $m$ disappears but $r$ contains a premium $r_{\mathrm{p}}$. Note that the meaning of $r$ here is different from what is originally used in the paper. Main banks will impose $r$ such that $q I(1+r)+(1-q) \operatorname{Min}\left\{A+I\left(1+r_{L}\right), I(1+r)\right\}-I=I r_{p}$ (instead of zero as in the original setting where rent extraction reflected as $m$ explicitly in (2)). Here rent extraction is modeled as main banks' behavior in setting interest rates, and rent extraction shows up in (2) implicitly through $r$.

Proposition 1: With bank financing only, the manager, on behalf of the shareholders, chooses the investment policy $\left[r_{H}^{e}\right]$, in which

$$
\begin{array}{ll}
r_{H}^{e}=\frac{r_{p}-(1-q) r_{L}}{q} & \text { if } \quad \frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \leq r_{L}, \\
r_{H}^{e}=\frac{(1-q) A}{q I}+r_{u} & \text { if } \quad-1<r_{L}<\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} . \tag{8}
\end{array}
$$

## Proof:

Purely based on $q I(1+r)+(1-q) \operatorname{Min}\left\{A+I\left(1+r_{L}\right), I(1+r)\right\}-I=I r_{p}$, main banks set the interest rate as follows: (Note that this has nothing to do with who makes corporate investment decisions.)

$$
\begin{align*}
& r=r_{p} \quad \text { if } \quad r_{L} \geq r_{p}-\frac{A}{I},  \tag{3}\\
& r=\frac{I r_{p}-(1-q)\left(A+I r_{L}\right)}{q I} \quad \text { if } \frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \leq r_{L}<r_{p}-\frac{A}{I},  \tag{4}\\
& r=r_{u} \quad \text { if }-1<r_{L}<\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} . \tag{5}
\end{align*}
$$

Managers, acting in the interests of the shareholders, will choose projects according to condition (2).

If $r_{L} \geq r_{p}-\frac{A}{I}$, according to (2) and (3), we have
$N P V_{\text {equity }}=q\left[A+I\left(r_{H}-r_{p}\right)\right]+(1-q)\left[A+I\left(r_{L}-r_{p}\right)\right]-A \geq 0$.
$N P V_{\text {equity }} \geq 0$ only if $r_{H} \geq \frac{r_{p}-(1-q) r_{L}}{q}$. Thus, $r_{H}^{e}=\frac{r_{p}-(1-q) r_{L}}{q}$, i.e. (7)
If $\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \leq r_{L}<r_{p}-\frac{A}{I}$, according to (2), we have $N P V_{\text {equity }}=q\left[A+I\left(r_{H}-r\right)\right]-A \geq 0$.
According to (4), we have
$N P V_{\text {equity }} \geq 0$ only if $r_{H} \geq \frac{r_{p}-(1-q) r_{L}}{q}$. Thus, again, $r_{H}^{e}=\frac{r_{p}-(1-q) r_{L}}{q}$, i.e. (7)
If $-1<r_{L}<\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I}$, according to (2) and (5), we have
$N P V_{\text {equity }}=q\left[A+I\left(r_{H}-r_{u}\right)\right]-A \geq 0$
$N P V_{\text {equity }} \geq 0$ only if $r_{H} \geq \frac{(1-q) A}{q I}+r_{u}$. Thus, $r_{H}^{e}=\frac{(1-q) A}{q I}+r_{u}$, i.e. (8).

Case 2: If the main bank can make corporate investment decisions, the payoff to banks is:

$$
\begin{align*}
N P V_{B a n k}= & q\left[\alpha\left[A+I\left(r_{H}-r\right)\right]+I(1+r)\right]+(1-q) \\
& {\left[\alpha \operatorname{Max}\left\{A+I\left(r_{L}-r\right), 0\right\}+\operatorname{Min}\left\{I(1+r), A+I\left(1+r_{L}\right)\right\}\right]-(\alpha A+I) \geq 0 } \tag{9}
\end{align*}
$$

In (9), $m$ disappears but rent extraction is implicit in $r$ which contains $r_{p}$.

Proposition 2: With bank financing only, the manager, on behalf of the bank, will choose the investment policy $\left[r_{H}{ }^{b}\right]$, in which

$$
\begin{align*}
& r_{H}^{b}=-\frac{(1-\alpha) r_{p}+\alpha(1-q) r_{L}}{\alpha q} \quad \text { if } r_{L} \geq \frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I},  \tag{11}\\
& r_{H}^{b}=-\frac{(1-\alpha)(1-q) A+(1-\alpha) q I r_{u}+(1-q) I r_{L}}{\alpha q I} \quad \text { if }-1<r_{L}<\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} . \tag{12}
\end{align*}
$$

## Proof:

If $r_{L} \geq r_{p}-\frac{A}{I}$, according to (3) and (9), we have

$$
\begin{aligned}
N P V_{\text {Bank }} & =q\left[\alpha\left[A+I\left(r_{H}-r_{p}\right)\right]+I\left(1+r_{p}\right)\right]+ \\
& (1-q)\left\{\alpha\left[A+I\left(r_{L}-r_{p}\right)\right]+I\left(1+r_{p}\right)\right\}-(\alpha A+I) \geq 0 .
\end{aligned}
$$

$N P V_{B a n k} \geq 0$ only if $r_{H}{ }^{b} \geq-\frac{(1-\alpha) r_{p}+\alpha(1-q) r_{L}}{\alpha q}$. Thus, $r_{H}{ }^{b}=-\frac{(1-\alpha) r_{p}+\alpha(1-q) r_{L}}{\alpha q}$, i.e. (11).

If $\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \leq r_{L}<r_{p}-\frac{A}{I}$, according to (9), we have

$$
N P V_{\text {Bank }}=q\left[\alpha\left[A+I\left(r_{H}-r\right)\right]+I(1+r)\right]+(1-q)\left[A+I\left(1+r_{L}\right)\right]-(\alpha A+I) \geq 0 .
$$

According to (4), we have
$N P V_{\text {Bank }} \geq 0$ only if $r_{H} \geq-\frac{(1-\alpha) r_{p}+\alpha(1-q) r_{L}}{\alpha q}$. Again, $r_{H}^{b}=-\frac{(1-\alpha) r_{p}+\alpha(1-q) r_{L}}{\alpha q}$, i.e.,
If $-1<r_{L}<\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I}$, according to (5) and (9), we have

$$
N P V_{\text {Bank }}=q\left[\alpha\left[A+I\left(r_{H}-r_{u}\right)\right]+I\left(1+r_{u}\right)\right]+(1-q)\left[A+I\left(1+r_{L}\right)\right]-(\alpha A+I) \geq 0
$$

$N P V_{B a n k} \geq 0$ only if $r_{H} \geq-\frac{(1-\alpha)(1-q) A+(1-\alpha) q I r_{u}+(1-q) I r_{L}}{\alpha q I}$. Thus,

$$
r_{H}^{b}=-\frac{(1-\alpha)(1-q) A+(1-\alpha) q I I_{u}+(1-q) I I_{L}}{\alpha q I}, \text { i.e., (12) }
$$

Fig. 1. Results of Propositions 1 and 2


It turns out that the results in Fig. 1 are similar to the original ones in the paper, except under- and overinvestment get severer for "safe" projects, i.e., when ( $r_{L}, r_{H}$ ) is close to the origin. The reason is that, in the case of overinvestment, banks have more incentive to launch projects with worse $\mathrm{r}_{\mathrm{L}}$ to get an unconditional $\mathrm{r}_{\mathrm{p}}$ than to get $m$ that is proportional only on $r_{H}$. In the case of underinvestment, managers have more incentive to skip projects with lower $\mathrm{r}_{\mathrm{H}}$ to avoid paying a constant $\mathrm{r}_{\mathrm{p}}$ than to avoid paying $m$ which is only proportional on $\mathrm{r}_{\mathrm{H}}$.

## Scenario II: Investment and debt-equity financing decisions under main bank control

The bank's total payoff including equity holdings in the firm:

$$
\begin{align*}
N P V_{b}= & q\left[\alpha\left(A+I\left(1+r_{H}\right)-D(1+r)\right)+D(1+r)\right]+(1-q) \\
& {\left[\alpha \operatorname{Max}\left\{A+I\left(1+r_{L}\right)-D(1+r), 0\right\}+\operatorname{Min}\left\{D(1+r), A+I\left(1+r_{L}\right)\right\}\right]-(\alpha A+\alpha e+D) } \tag{13}
\end{align*}
$$

The main bank will set an interest rate that contains $r_{p}$ such that
$q D(1+r)+(1-q) \operatorname{Min}\left\{A+I\left(1+r_{L}\right), D(1+r)\right\}-D=I r_{p}$.
Again, rent extraction is implicitly modeled in (13) through r.

Proposition 3: In the case of financing with new equity and debt, the manager, on behalf of the bank, will choose the optimal financing policy, $D^{*}$, and the investment policy $\left[r_{H}{ }^{b}\right]$, such that

$$
\begin{align*}
& D^{*}=I \quad \text { if } r_{L}>\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I}, \text { or } r_{L} \leq \frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \text { and } q+q r_{u}-1>0  \tag{19}\\
& D^{*}=\frac{(1-q)\left(A+I+I r_{L}\right)}{1-q-q r_{u}+r_{p}} \quad \text { if } r_{L} \leq \frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \text { and } q+q r_{u}-1<0 \text {, } \tag{20}
\end{align*}
$$

Corollary 3: In the case of financing with new equity and debt, when the bank requires a higher cutoff level, $X$, on its payoff, i.e., $N P V_{b} \geq X$, the manager, working on behalf of the bank, will choose the investment policy $\left[r_{H}{ }^{b X}\right]$, in which

$$
\begin{align*}
& r_{H}^{b x}=\frac{X-(1-\alpha) I r_{p}}{\alpha q I}-\frac{(1-q) r_{L}}{q} \quad \text { if } r_{L}>\frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I},  \tag{23}\\
& r_{H}^{b x}=\frac{X-(1-\alpha)(1-q) A-(1-\alpha) q I r_{u}}{\alpha q I}-\frac{(1-q) r_{L}}{\alpha q} \\
& \quad \text { if } r_{L} \leq \frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \text { and }\left(q+q r_{u}-1\right)>0  \tag{24a}\\
& r_{H}^{b x}=\frac{\left(1-q-q r_{u}+r_{p}\right) X-(1-\alpha)(1-q)(A+I) r_{p}}{\alpha q I\left(1-q-q r_{u}+r_{p}\right)}-\frac{(1-q)\left(r_{p}+\alpha-\alpha q-\alpha q r_{u}\right) r_{L}}{\alpha q\left(1-q-q r_{u}+r_{p}\right)}
\end{align*}
$$

$$
\begin{equation*}
\text { if } r_{L} \leq \frac{r_{p}-q r_{u}}{1-q}-\frac{A}{I} \text { and }\left(q+q r_{u}-1\right)<0 \tag{24b}
\end{equation*}
$$

Notice that

$$
-\frac{(1-q)\left(r_{p}+\alpha-\alpha q-\alpha q r_{u}\right) r_{L}}{\alpha q\left(1-q-q r_{u}+r_{p}\right)} \leq-\frac{(1-q)}{\alpha q} \leq-\frac{(1-q)}{q}
$$

## Proof:

From $q D(1+r)+(1-q) \operatorname{Min}\left\{A+I\left(1+r_{L}\right), D(1+r)\right\}-D=I r_{p}$, banks set the interest rate as follows.

$$
\begin{align*}
& r=r_{p} \quad \text { if } r_{L} \geq \frac{D\left(1+r_{p}\right)-A-I}{I},  \tag{16}\\
& r=\frac{D r_{p}+(1-q)\left\{D-\left[A+I\left(1+r_{L}\right)\right]\right\}}{q D} \\
& \text { if } \frac{(1-q)(D-I)-q D r_{u}+D r_{p}}{(1-q) I}-\frac{A}{I}<r_{L}<\frac{D\left(1+r_{p}\right)-A-I}{I},  \tag{17}\\
& r=r_{u} \quad \text { if } \quad r_{L} \leq \frac{(1-q)(D-I)-q D r_{u}+D r_{p}}{(1-q) I}-\frac{A}{I} . \tag{1}
\end{align*}
$$

Below we look at financing and investment decisions under conditions (16), (17) and (18), respectively.

If $r_{L} \geq \frac{D\left(1+r_{p}\right)-A-I}{I}$ (16), according to (13), we have

$$
\begin{align*}
N P V_{b} & =q\left[\alpha\left(A+I\left(1+r_{H}\right)-D\left(1+r_{p}\right)\right)+D\left(1+r_{p}\right)\right] \\
& +(1-q)\left[\alpha\left(A+I\left(1+r_{L}\right)-D\left(1+r_{p}\right)\right)+D\left(1+r_{p}\right)\right]-[\alpha A+\alpha I+(1-\alpha) D], \tag{44}
\end{align*}
$$

and

$$
\begin{equation*}
\frac{d N P V_{b}}{d D}=(1-\alpha) r_{p} \geq 0 \tag{45}
\end{equation*}
$$

Thus, $D^{*}=I$, because $N P V_{b}$ here is an increasing function of $D(\leq I)$.
If the bank requires a higher cutoff level, X , on its payoff, i.e.,

$$
\begin{aligned}
& N P V_{b}=q\left[\alpha\left(A+I\left(1+r_{H}\right)-D^{*}\left(1+r_{p}\right)\right)+D^{*}\left(1+r_{p}\right)\right] \\
& +(1-q)\left[\alpha\left(A+I\left(1+r_{L}\right)-D^{*}\left(1+r_{p}\right)\right)+D^{*}\left(1+r_{p}\right)\right]-\left[\alpha A+\alpha I+(1-\alpha) D^{*}\right] \geq X
\end{aligned}
$$

then, we have

$$
\begin{aligned}
& N P V_{b}=q\left[\alpha\left(A+I\left(1+r_{H}\right)-I\left(1+r_{p}\right)\right)+I\left(1+r_{p}\right)\right] \\
& +(1-q)\left[\alpha\left(A+I\left(1+r_{L}\right)-I\left(1+r_{p}\right)\right)+I\left(1+r_{p}\right)\right]-[\alpha A+\alpha I+(1-\alpha) I] \geq X
\end{aligned}
$$

Thus, $r_{H} \geq \frac{X-(1-\alpha) I r_{p}}{\alpha q I}-\frac{(1-q) r_{L}}{q}$, i.e., $r_{H}^{b x}=\frac{X-(1-\alpha) I r_{p}}{\alpha q I}-\frac{(1-q) r_{L}}{q}$

If $\frac{(1-q)(D-I)-q D r_{u}+D r_{p}}{(1-q) I}-\frac{A}{I}<r_{L}<\frac{D\left(1+r_{p}\right)-A-I}{I}$ (17), according to (13), we have $N P V_{b}=q\left[\alpha\left\{A+I\left(1+r_{H}\right)-D\left(1+\frac{D r_{p}+(1-q)\left[D-\left(A+I\left(1+r_{L}\right)\right]\right.}{q D}\right)\right\}\right.$

$$
\begin{equation*}
\left.+D\left(1+\frac{D r_{p}+(1-q)\left[D-\left(A+I\left(1+r_{L}\right)\right]\right.}{q D}\right)\right]+(1-q)\left[A+I\left(1+r_{L}\right)\right]-[\alpha A+\alpha I+(1-\alpha) D] \tag{46}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d N P V_{b}}{d D}=(1-\alpha) r_{p} \geq 0 \tag{47}
\end{equation*}
$$

Thus, $D^{*}=I$, because $N P V_{b}$ here is an increasing function of $D(\leq I)$.
If the bank requires a higher cutoff level, X , on its payoff, i.e.,

$$
\begin{aligned}
N P V_{b}= & q\left[\alpha\left\{A+I\left(1+r_{H}\right)-D^{*}\left(1+\frac{D^{*} r_{p}+(1-q)\left[D^{*}-\left(A+I\left(1+r_{L}\right)\right)\right]}{q D^{*}}\right)\right\}\right. \\
& \left.+D^{*}\left(1+\frac{D^{*} r_{p}+(1-q)\left[D^{*}-\left(A+I\left(1+r_{L}\right)\right)\right]}{q D^{*}}\right)\right]+(1-q)\left[A+I\left(1+r_{L}\right)\right]-\left[\alpha A+\alpha I+(1-\alpha) D^{*}\right] \geq X
\end{aligned}
$$

Since $D^{*}=I$, we have

$$
r_{H} \geq \frac{X-(1-\alpha) I r_{p}}{\alpha q I}-\frac{(1-q) r_{L}}{q} \text {,i.e., } r_{H}^{b x}=\frac{X-(1-\alpha) I r_{p}}{\alpha q I}-\frac{(1-q) r_{L}}{q}
$$

If $r_{L} \leq \frac{(1-q)(D-I)-q D r_{u}+D r_{p}}{(1-q) I}-\frac{A}{I}$ (18), according to (13), we have

$$
\begin{equation*}
N P V_{b}=q\left[\alpha\left(A+I\left(1+r_{u}\right)-D\left(1+r_{u}\right)\right)+D\left(1+r_{u}\right)\right]+(1-q)\left[A+I\left(1+r_{L}\right)\right]-[\alpha A+\alpha I+(1-\alpha) D] \tag{48}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d N P V_{b}}{d D}=(1-\alpha)\left(q+q r_{u}-1\right) \tag{49}
\end{equation*}
$$

Now we have to consider two cases.
Case 1: If $\left(q+q r_{u}-1\right)>0$, then $\frac{d N P V_{b}}{d D}=(1-\alpha)\left(q+q r_{u}-1\right)>0$. Thus, $D^{*}=I$, because $N P V_{b}$ here is an increasing function of $D(\leq I)$.

If the bank requires a higher cutoff level, X , on its payoff, i.e.,
$N P V_{b}=q\left[\alpha\left(A+I\left(1+r_{H}\right)-D^{*}\left(1+r_{u}\right)\right)+D^{*}\left(1+r_{u}\right)\right]+(1-q)\left[A+I\left(1+r_{L}\right)\right]-\left[\alpha A+\alpha I+(1-\alpha) D^{*}\right] \geq X$

Since $D^{*}=I$, we have
$r_{H} \geq \frac{X-(1-\alpha)(1-q) A-(1-\alpha) q I r_{u}}{\alpha q I}-\frac{(1-q) r_{L}}{\alpha q}$,
i.e., $r_{H}^{b x}=\frac{X-(1-\alpha)(1-q) A-(1-\alpha) q I r_{u}}{\alpha q I}-\frac{(1-q) r_{L}}{\alpha q}$

Case 2: If $\left(q+q r_{u}-1\right)<0$, then $\frac{d N P V_{b}}{d D}=(1-\alpha)\left(q+q r_{u}-1\right)<0$.
$N P V_{b}$ is a decreasing function of $D$. Given $r_{L} \leq \frac{(1-q)(D-I)-q D r_{u}+D r_{p}}{(1-q) I}-\frac{A}{I}$ (18),

$$
D^{*}=\frac{(1-q)\left(A+I+I r_{L}\right)}{1-q-q r_{u}+r_{p}}
$$

If the bank requires a higher cutoff level, X , on its payoff, i.e.,
$N P V_{b}=q\left[\alpha\left(A+I\left(1+r_{H}\right)-D^{*}\left(1+r_{u}\right)\right)+D^{*}\left(1+r_{u}\right)\right]+(1-q)\left[A+I\left(1+r_{L}\right)\right]-\left[\alpha A+\alpha I+(1-\alpha) D^{*}\right] \geq X$.

Since $D^{*}=\frac{(1-q)\left(A+I+I r_{L}\right)}{1-q-q r_{u}+r_{p}}$,
we have $r_{H} \geq \frac{\left(1-q-q r_{u}+r_{p}\right) X-(1-\alpha)(1-q)(A+I) r_{p}}{\alpha q I\left(1-q-q r_{u}+r_{p}\right)}-\frac{(1-q)\left(r_{p}+\alpha-\alpha q-\alpha q r_{u}\right) r_{L}}{\alpha q\left(1-q-q r_{u}+r_{p}\right)}$,
i.e.
$r_{H}^{b x}=\frac{\left(1-q-q r_{u}+r_{p}\right) X-(1-\alpha)(1-q)(A+I) r_{p}}{\alpha q I\left(1-q-q r_{u}+r_{p}\right)}-\frac{(1-q)\left(r_{p}+\alpha-\alpha q-\alpha q r_{u}\right) r_{L}}{\alpha q\left(1-q-q r_{u}+r_{p}\right)}$

Fig. 2. Results for Proposition 3


It turns out that while the original, strong curvature (caused by the asymmetric rent extraction, m) is gone in Figure 2, the main results on over- and underinvestment remain. Also note that the results under conditions $\left(q+q r_{u}-1\right)>0(24 \mathrm{a})$ and $\left(q+q r_{u}-1\right)<0$ (24b) are similar. The difference is trivial; for $r_{L}<\frac{r_{p}-q r_{u}}{(1-q)}-\frac{A}{I}$, those straight lines are steeper if $\left(q+q r_{u}-1\right)>0$ than if $\left(q+q r_{u}-1\right)<0$ (not shown in Figure 2).

## Scenario III: Firm controlled financing decisions under funding competition (either debt or equity)

A firm's cost of using bank loan becomes: $\mathrm{C}_{\mathrm{Bank}}=\mathrm{I} r_{p}$.
The firm will choose equity financing if $C_{\text {Equity }}<C_{\text {Bank }}$, namely

$$
\begin{equation*}
\beta\left\{A+I\left(1+q r_{H}+(1-q) r_{L}\right)\right\}-I<I r_{p} \tag{29}
\end{equation*}
$$

Recall $\beta$ is the share required by the new equity holders (see (28) in the paper), and the market expectation for $q$ is $E(q)=\left(q_{1}+q_{u}\right) / 2$.

Given $\mathrm{q}_{1}$, we have

$$
\begin{equation*}
q_{u}=\frac{E(q) I\left(r_{H}-r_{L}\right)+r_{p}\left(A+I(1+E(q)) r_{H}+(1-E(q)) r_{L}\right)}{I\left(r_{H}-r_{L}\right)} \tag{31,32}
\end{equation*}
$$

## Proposition 6:

When using either debt or new equity to finance a project, the manager, acting on behalf of the existing shareholders, prefers new equity over debt as long as $C_{\text {Bank }}>C_{\text {Equity, }}$, namely,

$$
\begin{equation*}
\left(\frac{A+I\left(1+q r_{H}+(1-q) r_{L}\right)}{\left.A+I\left(1+E[q] r_{H}+(1-E[q]) r_{L}\right)\right\}}-1\right)<r_{p} \tag{34,35}
\end{equation*}
$$

where

$$
\begin{equation*}
E(q)=\frac{q_{l} I\left(r_{H}-r_{L}\right)+\left(A+I+I r_{L}\right) r_{p}}{\left(1-r_{p}\right) I\left(r_{H}-r_{L}\right)} \tag{36,37}
\end{equation*}
$$

From Proposition 6, the decision rule to choose new equity instead of debt is:

$$
\begin{array}{ll}
r_{H}<\frac{\left(q_{l}+q_{l} r_{p}-q+q r_{p}-2 r_{p}\right) I r_{L}-2 r_{p}(A+I)}{\left(q_{l}+q_{l} r_{p}-q+q r_{p}\right) I} & \text { if } q_{l}+\left(q_{l}+q\right) r_{p}-q<0 \\
r_{H}>\frac{\left(q_{l}+q_{l} r_{p}-q+q r_{p}-2 r_{p}\right) I r_{L}-2 r_{p}(A+I)}{\left(q_{l}+q_{l} r_{p}-q+q r_{p}\right) I} & \text { if } q_{l}+\left(q_{l}+q\right) r_{p}-q>0 \tag{R2}
\end{array}
$$

## Proof:

The firm will choose equity financing if $\beta\left\{A+I\left(1+q r_{H}+(1-q) r_{L}\right)\right\}-I<I r_{p}$

$$
\text { i.e., } q<\frac{I\left(1+r_{p}\right)-\beta A-\beta I-\beta I r_{L}}{\beta I\left(r_{H}-r_{L}\right)}
$$

Since $\quad \beta=\frac{I}{A+I\left[1+E(q) r_{H}+(1-E(q)) r_{L}\right]}$

$$
\begin{equation*}
q_{u}=\frac{E(q) I\left(r_{H}-r_{L}\right)+r_{p}\left(A+I(1+E(q)) r_{H}+(1-E(q)) r_{L}\right)}{I\left(r_{H}-r_{L}\right)} \tag{31,32}
\end{equation*}
$$

Given that $q$ is uniformly distributed in [ $\mathrm{q}_{\mathrm{l}}, \mathrm{q}_{\mathrm{u}}$ ], the outside equity investors' expected payoffs will be:

$$
\begin{aligned}
E & =\int_{q_{l}}^{q_{u}} \frac{1}{q_{u}-q_{l}} \beta\left\{A+I+I\left[q r_{H}+(1-q) r_{L}\right]\right\} d q \\
& =\int_{q_{l}}^{q_{u}} \frac{1}{q_{u}-q_{l}}\left\{A+I+I\left[q r_{H}+(1-q) r_{L}\right]\right\} \frac{I}{A+I+I\left[E[q] r_{H}+(1-E[q]) r_{L}\right]} d q .
\end{aligned}
$$

A fair market price under risk neutrality makes the investors’ expected earnings exactly equal to their initial investment $I$. Thus, $E=I$. Solving it, we have $E[q]=\frac{q_{l}+q_{u}}{2}$. Considering (31, 32), we have

$$
\begin{equation*}
E(q)=\frac{q_{l} I\left(r_{H}-r_{L}\right)+\left(A+I+I r_{L}\right) r_{p}}{\left(1-r_{p}\right) I\left(r_{H}-r_{L}\right)} \tag{36,37}
\end{equation*}
$$

The firm will choose equity financing if $\beta\left\{A+I\left(1+q r_{H}+(1-q) r_{L}\right)\right\}-I<I r_{p}$
while $\beta=\frac{I}{A+I\left[1+E(q) r_{H}+(1-E(q)) r_{L}\right]}$ and $E(q)=\frac{q_{l} I\left(r_{H}-r_{L}\right)+\left(A+I+I r_{L}\right) r_{p}}{\left(1-r_{p}\right) I\left(r_{H}-r_{L}\right)}$
Thus the firm will choose equity financing when
$\left[q_{l}+\left(q_{l}+q\right) r_{p}-q\right] I r_{H}>\left(q_{l}+q_{l} r_{p}-q+q r_{p}-2 r_{p}\right) I r_{L}-2 r_{p}(A+I)$,
i.e. $\quad r_{H}<\frac{\left(q_{l}+q_{l} r_{p}-q+q r_{p}-2 r_{p}\right) I r_{L}-2 r_{p}(A+I)}{\left(q_{l}+q_{l} r_{p}-q+q r_{p}\right) I} \quad$ if $q_{l}+\left(q_{l}+q\right) r_{p}-q<0$ (R1)

$$
r_{H}>\frac{\left(q_{l}+q_{l} r_{p}-q+q r_{p}-2 r_{p}\right) I r_{L}-2 r_{p}(A+I)}{\left(q_{l}+q_{l} r_{p}-q+q r_{p}\right) I} \quad \text { if } q_{l}+\left(q_{l}+q\right) r_{p}-q>0 \text { (R2) }
$$

To understand the decision rule in (R1)-(R2), we need to consider three cases:

Case 1: $q_{l}+q_{l} r_{p}-q+q r_{p}<0$ (This condition is more likely when $r_{p}$ is lower.)

Here, the slope for $r_{L}$ in (R1), $1+\frac{-2 r_{p}}{q_{l}+q_{l} r_{p}-q+q r_{p}}$, is positive; and the intercept, $\frac{-2 r_{p}(A / I+1)}{q_{l}+q_{l} r_{p}-q+q r_{p}}$, is also positive. See Figure 4a. Note that the slope increases with $\mathrm{r}_{\mathrm{p}}$.

Figure 4a: when $r_{p}$ is low.


Thus, in general, debt is used to finance projects with high downside risk and new equity is used to finance projects with high growth potential but limited downside risk. If adverse shocks occur, banks suffer more, consistent with the result in the paper.

There is, however, a situation that may allow debt to finance projects with very high $\mathrm{r}_{\mathrm{H}}$ and very low $r_{L}$ at the same time. This is because bank costs here are fixed up front ( $=\mathrm{Ir}_{\mathrm{p}}$ ) instead of changing with $\mathrm{r}_{\mathrm{H}}$ as in the paper-a proportion, m , of $\mathrm{r}_{\mathrm{H}}$. This further demonstrates why the original setting of the paper more captures the true meaning of ex post rent extraction.

If ex post funding competition is keen, $\mathrm{r}_{\mathrm{p}}$ is likely to be low and hence the bank holdup behavior tends to be contained for high growth firms which can tap into new equity (see Wu, Sercu and Yao, 2009 for some empirical support in Japan for this view).

Case 2: $q_{l}+q_{l} r_{p}-q+q r_{p}>0$ (This becomes more likely when $r_{p}$ becomes higher.)
The condition in case 2 means that the slope for $\mathrm{r}_{\mathrm{L}}$ in (R2), $1+\frac{-2 r_{p}}{q_{l}+q_{l} r_{p}-q+q r_{p}}$, is negative, and the intercept, $\frac{-2 r_{p}(A / I+1)}{q_{l}+q_{l} r_{p}-q+q r_{p}}$, is negative. Note that the slope increases in $r_{p}$, or becomes more flat with an increase in $r_{p}$.

Case 2 has two situations:
Case 2a: $1+\frac{-2 r_{p}}{q_{l}+q_{l} r_{p}-q+q r_{p}}<-\frac{q}{1-q}$

This means that the slope is steeper than the slope for $r_{L}$ under the fist best rule, $-q /(1-q)$, as shown in Fig. 4b.
Figure 4b: when $r_{p}$ is high


This is the situation very similar to the result in the paper.

Case 2b: $1+\frac{-2 r_{p}}{q_{l}+q_{l} r_{p}-q+q r_{p}}>-\frac{q}{1-q}$

This means that the slope in (R2) is less steeper than the slope for $r_{L}$ under the fist best rule, $-q /(1-q)$. Given its negative intercept, this means a corner solution of all equity financing.

In all cases, the message is preserved that an increase in $r_{p}$ aggravates the bias "bank loan for downside risk and equity for upward potential" as shown in the paper.

End

