City University of Hong Kong

Course code & title:	MA6606 Computational Linear Algebra
Session:	Semester A, 1998-1999
Date:	January 6, 1999
Time:	6:30 pm — 9:30 pm
Time allowed:	Three hours

This paper has THREE pages. (Including this page)

Instructions to candidates:

- The paper has **SIX** questions.
- Attempt all **SIX** questions.
- All questions carry equal marks.
- Start each question on a new page.
- Show all working.

Materials, aids & instruments permitted to be used during examination:

- Non-programmable portable battery operated calculator.

1. Let A be the following symmetric matrix,

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Find a lower triangular matrix L and tridiagonal matrix T, such that $A = LTL^T$.

2. Find the QR factorization of the following matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -a & a \\ 0 & a & -a \end{bmatrix},$$

where a is a constant.

3. For the matrix

$$T = \begin{bmatrix} \epsilon & 1 & 0 \\ 1 & 0 & \epsilon \\ 0 & \epsilon & 0 \end{bmatrix},$$

carry out one iteration of the QR method (for symmetric matrix eigenvalue problem) with Wilkinson's shift.

4. Let T be the following symmetric tridiagonal matrix

$$T = \begin{bmatrix} a_1 & b_1 & & \\ b_1 & a_2 & \ddots & \\ & \ddots & \ddots & b_{n-1} \\ & & b_{n-1} & a_n \end{bmatrix}.$$

To calculate its determinant, we use the recurrence formula

$$\Delta_j = a_j \Delta_{j-1} - b_{j-1}^2 \Delta_{j-2}$$

where Δ_j is the determinant of $T(1:j, 1:j)$, i.e., the $j \times j$ matrix obtained from T by keeping the first j rows and the first j columns. Show that this leads to a backward stable algorithm for calculating $\det(T)$.

5. Let A be a real, symmetric, non-singular $m \times m$ matrix, and $A = QR$ be the QR factorization of A. Let z be the solution of $Az = (0, ..., 0, 1)^T$. Show that

$$q_m = \pm \frac{z}{||z||_2}$$

where q_m is the last column of Q.

6. Let A be a real symmetric matrix, λ_1, λ_2 be two of its eigenvalues and $\vec{v}_1$, $\vec{v}_2$ be the corresponding unit eigenvectors. The Lanczos method is used for the tridiagonal reduction

$$A\ [q_1, q_2, ...] = [q_1, q_2, ...] \begin{pmatrix} \alpha_1 & \beta_1 & \\ \beta_1 & \alpha_2 & \ddots \\ & \ddots & \ddots \end{pmatrix}$$

with the initial vector $q_1 = a\vec{v}_1 + b\vec{v}_2$ (where $a^2 + b^2 = 1$). Find the formulas for α_1, β_1 and α_2 in terms of λ_1, λ_2, a and b. Show that the eigenvalues of the 2×2 matrix $\begin{pmatrix} \alpha_1 & \beta_1 \\ \beta_1 & \alpha_2 \end{pmatrix}$ are λ_1 and λ_2.

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