Computing Optimal Waveguide Bends with Constant Width

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Abstract

A numerical method is developed for computing waveguide bends that preserve as much power in the fundamental mode as possible. The method solves an optimization problem for a small number of points used to define the bend axis by cubic spline functions. A wide-angle beam propagation method fomulated in a curvilinear coordinate system is used to compute the wave field in the bend. Comparing with a circular bend and an S-bend given by a cosine curve, the optimal bends have a smaller curvature near the two ends for better connection with the straight input and output waveguides. For multimode waveguides, the optimal bends can be used to remove the coupling between the fundamental mode and other propagating modes.

1 Introduction

Waveguide bends are important building blocks of integrated optical circuits [1]. They are used to change propagation directions or introduce lateral displacements, and they may serve other purposes such as polarization rotators [2]. A basic problem facing a bend is the power loss related to the open nature of optical waveguides. Passing through a bend, a part of the power carried by a propagating mode is consumed to excite the radiating waves. To keep the power loss at an acceptable level, the size of a conventional optical waveguide bend must be relatively large comparing with the wavelength. However, small bending structures are desired in integrated optics.

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Most studies on waveguide bends are restricted to circular bends with a constant curvature. In that case, the fundamental problem is to calculate the leaky modes of the bend. For two-dimensional step-index structures, this can be done analytically [3]. For three-dimensional waveguides, we can use rigorous numerical methods such as the finite difference method [4, 5], the finite element method [6] and the mode matching method [7], etc. Alternatively, we can use semi-analytic methods that involve some analytic approximations, such as those developed in [8] and [9]. For circular bends, it is also important to analyze the scattering at the interface between a straight and a bent waveguide. A small lateral offset of the waveguide axis can be used to reduce the transition loss at such an interface. Numerical methods for arbitrarily bent waveguides are also available [10, 11]. The eigenmode expansion method [12] can be used, if the structure is composed of segments each having a constant curvature. More generally, we can re-write the governing equation in a curvilinear coordinate system following the axis of the bend. A wide-angle beam propagation method (BPM) was developed [11] using such a coordinate system.

It is important to design compact waveguide bends with minimal power loss. Previous efforts have mostly been concentrated on circular bends or concatenation of circular bends [13]. The design of an arbitrarily bent waveguide was considered by Baets and Lagasse [10]. They have used a paraxial BPM based on an approximate equation derived for bends with small curvatures. In this paper, we develop an efficient method for computing optimal waveguide bends using a wide-angle BPM formulated in the curvilinear coordinate system along the bend axis. More precisely, given the two end points of a waveguide bend and the directions of the straight waveguides connecting it, we calculate the bend that preserves as much power in the fundamental mode as possible. The optimization is for a fixed wavelength and for bends with the same refractive index profile as the straight waveguide. In particular, the width of the waveguide core in the bend is the same as that of the connecting straight waveguides. The axis of the bend is given by clamped cubic spline functions constructed from a small set of points. We use an optimization scheme to calculate these points so that the power loss is minimized. Our method is illustrated by computing an optimal 90^o bend and an S-bend for $15\,\mu m$ lateral displacement.

2 The beam propagation method

For transverse electric (TE) waves in two-dimensional structures, the y-component of the electric field, denote by u in this paper, satisfies

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + k_0^2 n^2 u = 0, \tag{1}$$

where k_0 is the free-space wavenumber and $n = n(x, y)$ is the refractive index function. For a bent waveguide, we assume that the waveguide axis is given by

$$z = f(s), \quad x = g(s), \tag{2}$$

where s is the arclength. For $s < 0$, the waveguide is straight and it is along the z-axis, therefore, $f(s) = s$ and $g(s) = 0$. The actual bend is given in the finite interval $0 < s < \Gamma$, where Γ is the total length of the bend. For $s > \Gamma$, the waveguide is again straight. We assume that the angle between its axis and the z-axis is Θ. Therefore, for $s > \Gamma$, we have

$$\begin{aligned} f(s) &= f(\Gamma) + (s - \Gamma)\cos\Theta, \\ g(s) &= g(\Gamma) + (s - \Gamma)\sin\Theta. \end{aligned}$$

A curvilinear coordinate system can be developed using s and ξ, where ξ is the coordinate in a direction normal to the waveguide axis. Since $(-g'(s), f'(s))$ is a unit vector normal to the waveguide axis, the point (z, x) is given by (s, ξ) satisfying

$$z = f(s) - \xi g'(s), \quad x = g(s) + \xi f'(s). \tag{3}$$

Using the above transformation, we obtain

$$r\frac{\partial}{\partial s}\left(\frac{1}{r}\frac{\partial u}{\partial s}\right) + r\frac{\partial}{\partial \xi}\left(r\frac{\partial u}{\partial \xi}\right) + r^2 k_0^2 n^2 u = 0, \tag{4}$$

where

$$r = 1 - \xi\kappa(s), \quad \kappa(s) = f'(s)g''(s) - f''(s)g'(s).$$

In the above, $\kappa(s)$ is the curvature of the axis at s. In principle, our problem is to solve Eq. (4) assuming that an incident field is given for $s < 0$ and only outgoing waves exist for $s > \Gamma$. Typically, the incident field is the fundamental propagating mode of the straight waveguide for $s < 0$ and we are interested in the transmitted field for $s \geq \Gamma$. If the semi-infinite segments for $s < 0$ and $s > \Gamma$ are identical single mode waveguides and $\phi_1(\xi)$ is the profile of the fundamental mode, we are concerned with $|T_1|^2$, where T_1 is the coefficient of ϕ_1 when the transmitted field $u(\Gamma, \xi)$ is expanded in the eigenmodes.

Since a typical waveguide bend does not change much on the scale of a wavelength, the BPM is applicable and efficient. A wide-angle BPM was developed in [11] based on the transformed Helmholtz equation (4). Assuming that wave field is dominated by its forward component in the increasing s direction, Eq. (4) is approximated by the following one-way Helmholtz equation

$$\frac{\partial u}{\partial s} = ik_0 n_* \sqrt{1 + X}\, u, \tag{5}$$

where n_* is a reference refractive index and X is an operator defined by

$$r\frac{\partial}{\partial \xi}\left(r\frac{\partial}{\partial \xi}\right) + r^2 k_0^2 n^2 = k_0^2 n_*^2 (1+X).$$

For a step from s_j to $s_{j+1} = s_j + \Delta s$, the field u satisfying (5) can be marched as

$$u_{j+1} = P(X)\, u_j, \quad P(X) = \exp\left[ik_0 n_* \Delta s\sqrt{1+X}\right], \tag{6}$$

where u_j and u_{j+1} approximate u at s_j and s_{j+1}, X is evaluated at $s_j + \Delta s/2$ and $P(X)$ is the one-way propagator. If for some integer p, we approximate the one-way propagator by

$$P(X) \approx a_0 + \sum_{l=1}^{p} \frac{a_l}{X + b_l},$$

where the coefficients a_0, a_l and b_l depend on p and $k_0 n_* \Delta s$, then (6) is further approximated by

$$u_{j+1} = a_0 u_j + \sum_{l=1}^{p} a_l w_l,$$

where w_l has to be solved from $(X + b_l)w_l = u_j$. Rational approximants for the one-way propagator P that can suppress the evanescent modes have been developed in [14] and [15].

3 Optimization problem

We are concerned with the design of a bend connecting two semi-infinite straight waveguides with identical refractive index profiles. In the zx-plane, the axis of the first waveguide is the negative z-axis and the origin is its endpoint. The axis of the second waveguide is the straight line that starts at (z_*, x_*) and forms an angle Θ with the z-axis. The bend is assumed to connect these two semi-infinite waveguides smoothly. Between these two endpoints, the axis of the bend is assumed to pass through m controlling points: (z_k, x_k) for $k = 1, 2, ..., m$. Each of these points can vary following a parameter. More precisely, the point (z_k, x_k) is allowed to change in the direction $\mathbf{q}_k$ near the point $\mathbf{p}_k$. That is,

$$(z_k, x_k) = \mathbf{p}_k + \eta_k \mathbf{q}_k, \quad k = 1, 2, ..., m, \tag{7}$$

where η_k is a scalar parameter, $\mathbf{q}_k$ is a unit vector and $\mathbf{p}_k$ is one of the points that specifies an initial guess of the optimal bend.

Let $\mathbf{p}_0 = (z_0, x_0) = (0, 0)$ and $\mathbf{p}_{m+1} = (z_{m+1}, x_{m+1}) = (z_*, x_*)$ be the two endpoints of the bend, we define d_k by

$$d_0 = 0, \quad d_{k+1} = d_k + |\mathbf{p}_{k+1} - \mathbf{p}_k|, \quad k = 0, 1, ..., m,$$

then the axis of the bend is specified by two cubic spline functions in terms of a parameter τ. More precisely, the waveguide axis is given by

$$z = F(\tau), \quad x = G(\tau), \quad 0 \le \tau \le d_{m+1}, \tag{8}$$

where F and G are piecewise cubic polynomials of τ (with continuous second order derivatives) satisfying

$$\begin{aligned} &F(d_k) = z_k, \quad G(d_k) = x_k, \quad k = 0, 1, ..., m+1, \\ &F'(d_0) = 1, \quad F'(d_{m+1}) = \cos\Theta, \\ &G'(d_0) = 0, \quad G'(d_{m+1}) = \sin\Theta. \end{aligned}$$

The cubic spline functions can be solved by standard methods described in textbooks on numerical analysis.

In section 2, the BPM is formulated for bent waveguides where the axis is given as functions of its arclength s in Eq. (2). Since the parameter τ in Eq. (8) is not the arclength, a transform is needed. The arclength of the bend axis is related to τ by

$$s = s(\tau) = \int_0^\tau \left\{[F'(\tau)]^2 + [G'(\tau)]^2\right\}^{1/2} d\tau. \tag{9}$$

In particular, it is necessary to calculate the total arclength of the bend

$$\Gamma = s(d_{m+1}) = \int_0^{d_{m+1}} \left\{[F'(\tau)]^2 + [G'(\tau)]^2\right\}^{1/2} d\tau.$$

The functions f and g are related to F and G by

$$f(s) = F(\tau), \quad g(s) = G(\tau),$$

where s is related to τ as in (9). This allows us to evaluate the derivatives of f and g in terms of those for F and G. We have

$$f'(s) = \frac{F'}{[(F')^2 + (G')^2]^{1/2}}, \quad f''(s) = \frac{G'[F''G' - F'G'']}{[(F')^2 + (G')^2]^2},$$

where the derivatives of F and G are evaluated at the corresponding τ. The derivatives of g have similar formulas. Furthermore, the curvature at s is given by

$$\kappa(s) = \frac{F'G'' - G'F''}{[(F')^2 + (G')^2]^{3/2}}.$$

Although the BPM can use a different step size Δs in each step, a constant step is more convenient, since then the rational approximant for $P(X)$ needs to be calculated only once. We assume that a total of M steps are needed for the BPM to propagate the field through the bend, thus

$$\Delta s = \frac{\Gamma}{M}, \quad s_j = j\Delta s, \quad j = 0, 1, ..., M.$$

Notice that for the step from s_j to s_{j+1}, the operator X is evaluated at $s_{j+1/2} = s_j + \Delta s/2$. Therefore, we need to calculate both τ_j and $\tau_{j+1/2}$ corresponding to s_j and $s_{j+1/2}$, respectively. Assuming that τ_j is already calculated, we can solve $\tau_{j+1/2}$ from

$$\int_{\tau_j}^{\tau_{j+1/2}} \left\{ [F'(\tau)]^2 + [G'(\tau)]^2 \right\}^{1/2} d\tau = \frac{\Delta s}{2}. \tag{10}$$

This equation can be solved iteratively using a nonlinear equation solver, such as Newton's method, together with a numerical approximation of the integral. For Newton's method, the derivative with respect to $\tau_{j+1/2}$ is readily available. To ensure convergence, we have used a hybrid method that combines the bisection method and Newton's method. Similarly, if $\tau_{j+1/2}$ is available, we can solve τ_{j+1}.

For our design problem, the two semi-infinite straight waveguides have identical refractive index profiles. We assume that the refractive index profile of the bend is also the same under the curvilinear coordinate system. Therefore, the refractive index function n in Eq. (4) is actually s-independent. For step-index waveguides, this implies that the width of the waveguide core is constant for the whole structure. Let ϕ_1 be the fundamental mode of the straight waveguide (for both $s < 0$ and $s > \Gamma$), the BPM is initialized with

$$u\big|_{s=0} = \phi_1.$$

After we obtain the field at $s = \Gamma$, we calculate transmission coefficient T_1 given by

$$T_1 = \frac{\int u(\Gamma, \xi)\phi_1(\xi)\, d\xi}{\int \phi_1^2(\xi)\, d\xi}.$$

Clearly, the coefficient T_1 is related to the positions of (z_k, x_k) for $1 \le k \le m$. Therefore, $T_1 = T_1(\eta_1, \eta_2, ..., \eta_m)$, where η_k specifies (z_k, x_k) as in (7). The power carried by the fundamental mode is proportional to $|T_1|^2$. For single mode waveguide, the power loss in decibels is given by $-20\log_{10}|T_1|$. For a multimode waveguide, some part of the power in the fundamental mode will be transferred to other propagating modes. To find the bend that preserves as much power in the fundamental mode as possible, we solve the optimization problem:

$$\max_{\eta_1,\dots,\eta_m} \; |T_1(\eta_1, \eta_2, ..., \eta_m)|^2. \tag{11}$$

4 Optimal 90° bend

In this section, we apply the method developed in previous sections to find an optimal 90° bend for a symmetric slab waveguide. The waveguide core of the entire structure, including the bend, has a constant width d. In the curvilinear coordinates s and ξ, the refractive index function is s-independent and it is given as

$$n(s, \xi) = \begin{cases} n_{co} & \text{for } |\xi| < d/2, \\ n_{cl} & \text{for } |\xi| > d/2. \end{cases} \tag{12}$$

In the following, we assume $n_{co} = 3.24$ and $n_{cl} = 3.17$.

We start with a circular bend whose axis is specified by

$$z = R\sin(s/R), \quad x = R\,[1 - \cos(s/R)]$$

for $0 < s < L = \pi R/2$, where R is the radius of the circular bend, s is the arclength and L is the total length of the bend. The axis of the straight waveguide for $s < 0$ is the negative z-axis. The straight waveguide for $s > L$ starts at (R, R) and its axis is parallel to the x-axis. The propagation of light through a circular bend can be analyzed rigorously by the eigenmode expansion method [12]. Although the propagating modes in the straight waveguide and the leaky modes in the circular bend can be found analytically [3], a numerical method is still needed to accurately model the coupling with the radiation modes. For that purpose, the transverse variable ξ can be truncated using perfectly matched layers (PMLs) [16, 17]. This gives rise to a discrete sequence of modes for both the straight waveguide and the circular bend. Let $\{\phi_j\}$ and $\{\varphi_j\}$ be the modes of the straight waveguide ($s < 0$ or $s > L$) and the circular bend, respectively, and β_j and γ_j be the corresponding propagation constants, we can expand the field as

$$u = \begin{cases} \phi_1 e^{i\beta_1 s} + \sum_{j=1}^{\infty} R_j \phi_j e^{-i\beta_j s}, & s < 0; \\ \sum_{j=1}^{\infty} \left[A_j \varphi_j e^{i\gamma_j s} + B_j \varphi_j e^{-i\gamma_j (s-L)} \right], & 0 < s < L; \\ \sum_{j=1}^{\infty} T_j \phi_j e^{i\beta_j (s-L)}, & s > L, \end{cases}$$

where an incident field corresponding to the fundamental propagating mode ϕ_1 of the straight waveguide is given for $s < 0$, $\{R_j\}$ and $\{T_j\}$ are the (amplitude) reflection and transmission coefficients. To be more specific, we normalize the modes such that

$$\int |\phi_j|^2 d\xi = \int |\varphi_j|^2 d\xi = 1. \tag{13}$$

After a truncation of the infinite sums, we can solve the coefficients $\{R_j, A_j, B_j, T_j\}$ from a set of equations established from the continuities of u and $\partial_s u$ at $s = 0$ and $s = L$.

For $d = 3\,\mu$m and the free-space wavelength $\lambda = 1.55\,\mu$m, the slab waveguide has three propagating modes. In Fig. 1, we show the power transmission coefficients $|T_1|^2$, $|T_2|^2$, $|T_3|^2$ and the power loss

$$\alpha = -10\log_{10}\left(|T_1|^2 + \frac{\beta_2}{\beta_1}|T_2|^2 + \frac{\beta_3}{\beta_1}|T_3|^2 \right).$$

The above definition of the power loss is related to fact that the power carried by a TE propagating mode $\{\phi_j, \beta_j\}$ is proportional to $\beta_j \int |\phi_j|^2 d\xi$ and the power carried by the three propagating modes in the transmitted field is proportional to $\sum_{j=1}^{3} |T_j|^2 \beta_j \int |\phi_j|^2 d\xi$. It is clear that a significant part of the power carried by the first mode is transferred

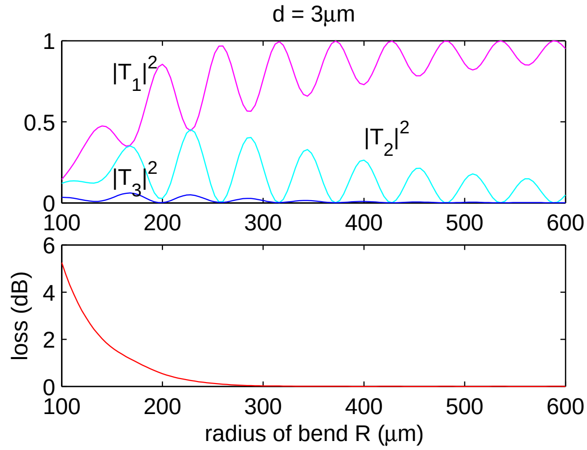

Figure 1: Power transmission coefficients and the power loss versus the radius of a circular bend for a multimode waveguide.

to the second mode. The third mode also carries a small amount of the total power. If the radius is small (say $R < 200\,\mu$m), the loss (due to coupling to the radiation modes and reflections) is significant. As R is increased, the power loss decreases to zero monotonically. In the output waveguide ($s > L$), the power distribution among the three propagating modes varies with the radius R in an oscillatory fashion, but the first mode has the general tendency of retaining more power as R is increased. The results shown in Fig. 1 are calculated using the eigenmode expansion method with ξ truncated to $-16.5\,\mu\text{m} < \xi < 6\,\mu$m. Perfectly matched layers of thickness $1.5\,\mu$m and $1\,\mu$m are used at the negative and positive ends of the ξ interval, respectively. The modes $\{\phi_j\}$ and $\{\varphi_j\}$ are calculated numerically following a finite difference discretization of the transverse operator with the grid size $\Delta\xi = 0.05\,\mu$m.

The results for the circular bend obtained by the eigenmode expansion method allow us to validate the BPM presented in section 2. In Fig. 2, we compare the power transmission coefficient $|T_1|^2$ calculated by these two methods. It is clear that BPM results are accurate, especially when R is large. The BPM solutions are obtained with the same discretization for ξ, a reference refractive index $n_* = 3.2$, a step size $\Delta s \approx 1.05\,\mu$m and the [3/4] Padé approximant of the one-way propagator $P(X)$.

To find a 90° bend that preserves the power in the fundamental mode as much as possible, we choose m points $\{\mathbf{p}_k\}$ from the circular bend and m unit vectors $\{\mathbf{q}_k\}$ by

$$\mathbf{p}_k = (\,R\sin\theta_k,\; R(1-\cos\theta_k)\,)\,, \quad \mathbf{q}_k = (\,\sin\theta_k,\; -\cos\theta_k\,)\,,$$

for $\theta_k = k\pi/[2(m+1)]$ and $k = 1, 2, ..., m$. They are used to define the m controlling points on the bend axis as given in (7). We have calculated the optimal bend using $m = 4$ for different values of R. The power transmission coefficient $|T_1|^2$ for the optimized bend is shown in Fig. 2 as the red curve. It is clear that coupling to the second mode is

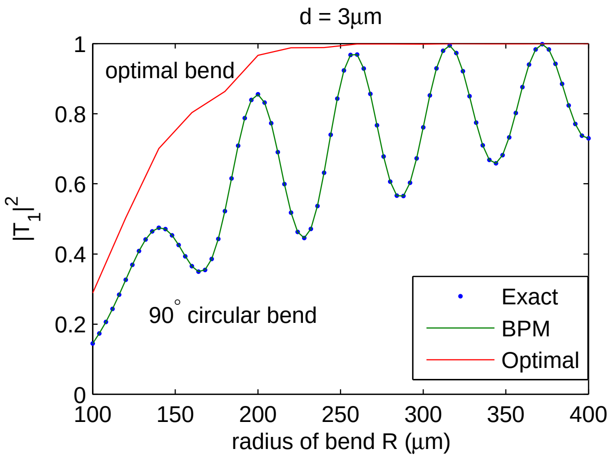

Figure 2: Power transmission coefficient $|T_1|^2$ of a circular bend and an optimized bend for a multimode waveguide.

significantly reduced and the oscillatory dependence of $|T_1|$ on R is avoided. In Fig. 3, we compare the axis of the circular bend and the optimized bend for $R = 275\,\mu$m. We

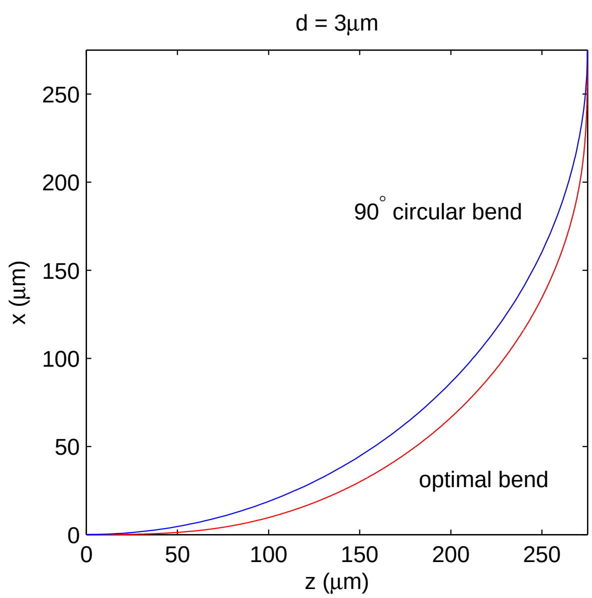

Figure 3: Axes of the circular bend and the optimal bend for a multimode waveguide.

observe that the optimal bend has a smaller curvature near the two ends, so that it has a better connection with the straight input and output waveguides, and it has a larger curvature than the the circular bend in the central part of the structure.

If the width of the core is reduced to $d = 1\,\mu$m, the waveguide has only one propagating mode. As shown in Fig. 4, the power loss defined as $\alpha = -20 \log_{10} |T_1|$ decreases monotonically as R is increased. We have also calculated the optimal 90° bend using $m = 4$ controlling points. The power loss of the optimal bend is also shown in Fig. 4

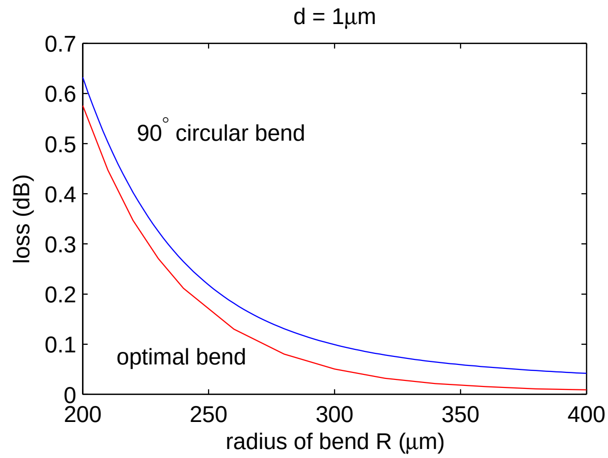

Figure 4: Power loss of a circular bend and an optimized bend for a single mode waveguide.

as the red curve. Unfortunately, the improvement for small values of R is quite limited. For larger R, the power loss of the optimal bend is significantly lower than the circular bend. In Fig. 5, the axes of the optimal bend and the circular bend are compared. Once

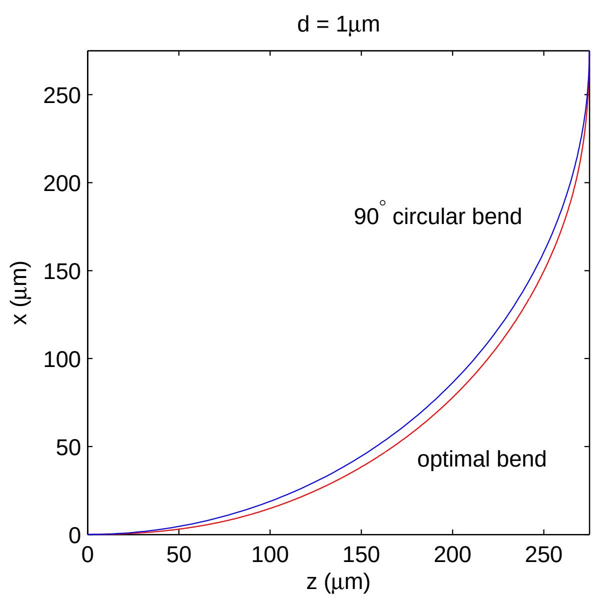

Figure 5: Axis of a circular bend and an optimized bend for a single mode waveguide.

again, we observe that the curvature of the optimal bend is smaller near the two ends of the bend.

5 Optimal S-bend

In this section, we consider an S-bend that introduces a lateral displacement of V within a longitudinal distance of W. As before, we assume that the axis of the input waveguide is the negative z-axis. The axis of the output waveguide is parallel to the z-axis and it is given by $x = V$ and $z > W$. As in the previous section, we assume that the input and output waveguides are two-dimensional symmetric slab waveguides with a core width d. The bend that connects these two straight waveguide is assumed to have the same core width and the same refractive index profile. In the curvilinear coordinate system (s, ξ), the refractive index function is then given in Eq. (12) with $n_{co} = 3.24$ and $n_{cl} = 3.17$.

Our starting point is the S-bend given by a cosine function:

$$z = F_0(\theta) = \frac{\theta}{\pi} W, \quad x = G_0(\theta) = \frac{V}{2}(1 - \cos\theta)$$

for $0 < \theta < \pi$. For such a structure, the eigenmode expansion method [12] is not so convenient. We can try to approximate the bend by segments with constant curvatures, but the interface between two nearby segments with different curvatures may be difficult to define [11]. On the other hand, the BPM has no difficulty. Since the bend axis is given in terms of the parameter θ, instead of the arclength s, the BPM must be implemented with some additional calculations as described in section 3. In particular, we need to calculate the total length of the bend

$$L = \int_0^\pi \left\{ [F_0'(\theta)]^2 + [G_0'(\theta)]^2 \right\}^{1/2} d\theta,$$

and to find θ_j and $\theta_{j+1/2}$ in connection with the discrete values $s_j = j\Delta s$ and $s_{j+1/2} = (j + 0.5)\Delta s$. Here, the step size $\Delta s = \Gamma / M$, where M is the total number of steps.

To calculate the optimal S-bend, we choose m controlling points given by

$$\begin{aligned} z_k &= \frac{kW}{m+1}, \\ x_k &= \frac{V}{2}\left[1 - \cos\left(\frac{k\pi}{m+1}\right)\right] + \eta_k, \end{aligned}$$

for $k = 1, 2, ..., m$. In terms of $\mathbf{p}_k$ and $\mathbf{q_k}$ as in (7), we have chosen the m points $\{\mathbf{p}_k\}$ from the cosine S-bend with their z-coordinates equally spaced. The vector $\mathbf{q}_k$ is the unit vector in the positive x-direction, i.e. $(0, 1)$ in the zx-plane. As before, the controlling points are used to define the axis of the bend and the parameters $\{\eta_k\}$ are optimized such that the power transmission coefficient $|T_1|^2$ of the fundamental mode is maximized.

For a fixed lateral displacement $V = 15\,\mu$m, we first consider S-bends for a waveguide with a core width $d = 3\,\mu$m. In Fig. 6, we show the power transmission coefficient $|T_1|^2$ as a function of longitudinal distance W for waves with a free space wavelength $\lambda = 1.55\,\mu$m. Since the corresponding straight waveguide has three propagating modes,

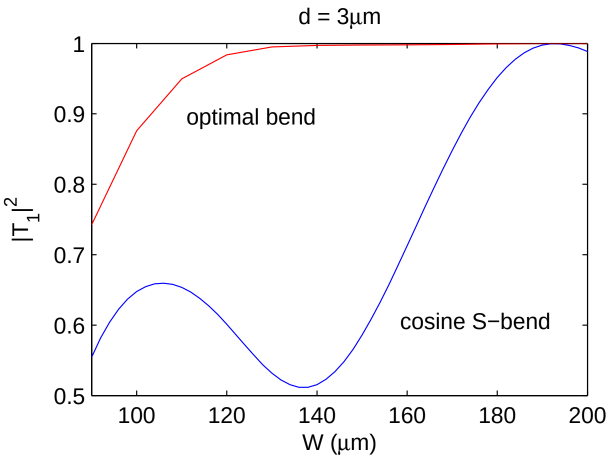

Figure 6: Power transmission coefficient $|T_1|^2$ of a cosine S-bend and an optimized S-bend for a multimode waveguide.

the cosine S-bend introduces a strong coupling between these modes. As indicated by the blue curve in Fig. 6, the magnitude of the transmission coefficient T_1 has large and oscillatory variations with W for the cosine S-bend. We have also calculated the optimal bend with $m = 5$ controlling points. The results are shown as the red curve in Fig. 6. We observe that for the optimal S-bend, $|T_1|$ increases with W monotonically. This indicates that the optimal S-bend can remove the coupling with the second and third propagating modes. The axes of the cosine S-bend and the optimal S-bend are compared in Fig. 7 for $W = 130\,\mu$m. We observe that the optimal S-bend has a smaller curvature near the

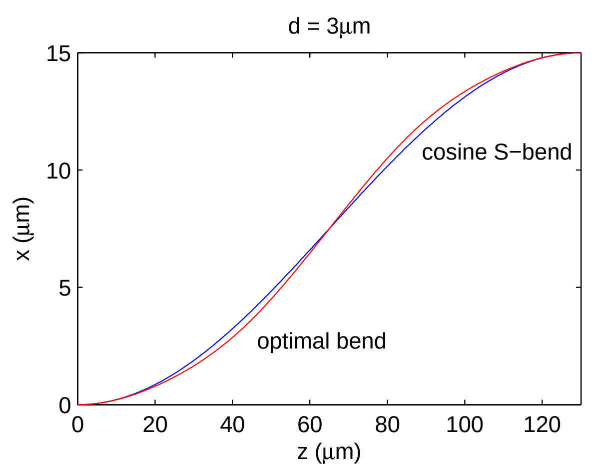

Figure 7: Axes of a cosine S-bend and an optimized S-bend for a multimode waveguide.

two ends and it varies more rapidly at the center. In fact, this is true for other values of W. These results are obtained by the BPM with the transverse variable truncated to $|\xi| < 11\,\mu$m and discretized with $\Delta\xi = 0.05\,\mu$m. Perfectly matched layers of thickness $1\,\mu$m are used at both ends of the ξ interval. The number of steps M varies for different values of W and the step size Δs varies from $1.003\,\mu$m to $1.017\,\mu$m. As before, we have used a reference refractive index $n_* = 3.2$ and a [3/4] Padé approximant for the one-way propagator $P(X)$.

Next, we calculate the optimal S-bend for the single mode slab waveguide with core width $d = 1\,\mu$m. All other parameters remain unchanged. For comparison, we also consider an S-bend given by a polynomial of degree 5 that connects the input and output waveguides smoothly up to the second order derivative:

$$z = W\tau, \quad x = V\tau^3(6\tau^2 - 15\tau + 10), \quad 0 < \tau < 1.$$

In Fig. 8, we compare the power loss $\alpha = -20\log_{10}|T_1|$ for the cosine S-bend, the fifth

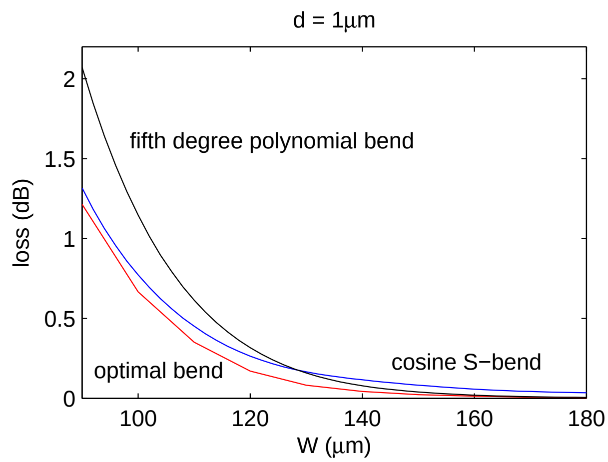

Figure 8: Power loss of a cosine S-bend and an optimized S-bend for a single mode waveguide.

degree polynomial bend, and the optimal S-bend. Same as the case of the 90° bend, it is difficult to reduce the power loss when W is small. For larger W, the fifth degree polynomial bend performs better than the cosine S-bend, although it is still not as good as the optimal bend. Compared with the cosine S-bend, the optimal bend has a much smaller power loss for relatively larger values of W. In Fig. 9. we compare the axes of the cosine S-bend, the fifth degree polynomial bend and the optimal S-bend for $W = 130\,\mu$m. We observe that the axis of the optimal bend stays somewhere in between those of the cosine S-bend and the fifth degree polynomial bend. Near the two ends of the bend, a small curvature seems to give better connections to the straight input and output waveguides, but that of the fifth degree polynomial is too small.

6 Conclusion

In this paper, we have developed a practical procedure for calculating waveguide bends that preserve as much power in the fundamental mode as possible. This is formulated as

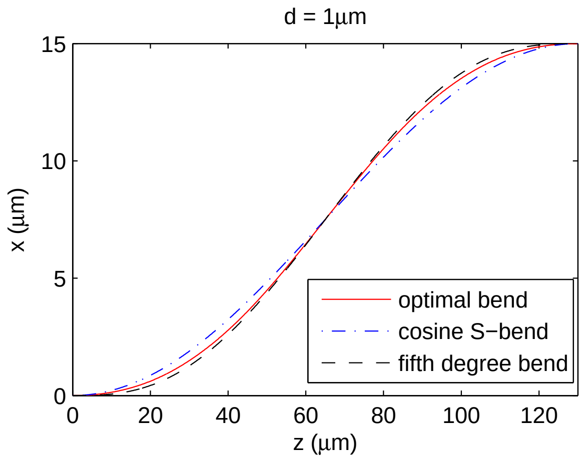

Figure 9: Axes of a cosine S-bend and an optimized S-bend for a single mode waveguide.

an optimization problem for a small number of points that define the bend axis using cubic spline functions. The propagation of the wave field in the bend is calculated by a wide-angle BPM formulated in a curvilinear coordinate system [11]. For bent waveguides with relatively small curvatures, the BPM gives accurate results as indicated in a comparison with the more rigorous eigenmode expansion method for a circular bend. The BPM is used, since it is much more efficient than the eigenmode expansion method for arbitrarily bent waveguides.

As examples, we have calculated optimal 90° bends and optimal S-bends for symmetric slab waveguides. For multimode waveguides, the optimal bends can significantly reduce the coupling between the fundamental mode and other propagating modes. For single mode waveguides, we observe that it is difficult to reduce the power loss if the size of the bend is small. However, a significant reduction of power loss can be achieved for relatively large bends. In all cases, we observe that the optimal bends tend to have small curvatures near the two ends. This provides a better connection between the bend and the straight input and output waveguides.

We have presented our method for TE waves in two-dimensional waveguide structures. The case for transverse magnetic polarization is similar. The refractive index is assumed to be s-independent in a curvilinear coordinate system that follows the axis of the bend. This implies that the width of the waveguide core is a constant in the bend. The method can be extended to bends with a variable core width, but two sets of points are needed to define the boundaries of the core. In principle, the method can be extended to three-dimensional bent waveguides laid on a substrate. However, wide-angle BPMs for three-dimensional wave-guiding structures are not very efficient and the paraxial BPM may have insufficient accuracy.

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