Improved bends for two-dimensional photonic crystal waveguides

Zhen Hu [a,*], Ya Yan Lu [b]

[a] *Department of Mathematics, Hohai University, Nanjing, Jiangsu, China*
[b] *Department of Mathematics, City University of Hong Kong, Kowloon, Hong Kong*

Abstract

For two-dimensional photonic crystals involving infinitely long dielectric rods or air-holes on square or triangular lattices, a number of high performance 60° and 90° waveguide bends are obtained by solving optimization problems involving the radii of a few rods or air-holes as the degrees of freedom. In particular, the proposed 60° bends significantly outperform previous designs that insert three or five identical air-holes in the bend. The optimization problems are solved using a recently developed method based on the so-called Dirichlet-to-Neumann (DtN) maps of the unit cells.

Keywords: Optical waveguides, photonic crystal waveguides, waveguide bends, optimal design, numerical methods.

1 Introduction

For high density photonic integration, sharp waveguide bends with low loss over a sufficiently large frequency range are essential. Photonic crystals (PhCs) [1] are excellent material to realize such waveguide bends. Mekis *et al.* [2] first theoretically demonstrated high transmission in a sharp 90° bend, where the background is an ideal two-dimensional (2D) PhC composed of circular dielectric rods on a square lattice. Over the years, many photonic structures and devices, including waveguide bends, have been designed and fabricated on the more realistic PhC slabs [3–6]. A PhC slab is a three-dimensional (3D) structure with a 2D periodicity. Typically, it is a dielectric slab with a triangular lattice

*Corresponding author. Tel.: +852 27887436; fax: +852 27887446. *E-mail address:* mayylu@cityu.edu.hk (Y. Y. Lu).

of circular air-holes. For waveguide bends in such air-hole-type PhCs, it is more difficult to realize high transmission with a wide bandwidth. Therefore, new designs with improved performance are still desired.

In the last decade, many different designs for PhC waveguide bends have appeared in the literature. The simplest designs involve only the radii of a few cylinders (dielectric rods or air-holes, including additional ones) around the center of the bend [7–10], where the centers of the cylinders remain on the original lattice. Alternative designs have been developed based on changing the positions of a few cylinders at the center [11–13]. The performance of bends can be further improved if both radii and positions of the cylinders are allowed to vary [14–17]. High performance bends can be designed if one gives up the circular geometry of the air-holes or dielectric rods [18–21], but the more complicated shapes also increase the difficulty of fabrication. Finally, waveguide bends have also been designed based on changing the lattice structure of the background PhC [22,23], but they affect a larger area around the bend and are not as suitable for photonic integration.

In this paper, based on an efficient numerical method developed in [24], we optimize waveguide bends in ideal 2D PhCs for the simplest case of varying only the radii of a few cylinders. For both 60° and 90° bends, we obtain new designs that have better performance than those given in [7] and [8]. In particular, for 60° bend in a PhC with a triangular lattice of air-holes, we have a very simple design with just two additional air-holes in the bend and its transmittance is more than 98.5% in a sufficiently large frequency interval.

2 Computation methods

Optimizing a waveguide bend in a realistic PhC slab is very challenging computationally. The optimization process typically requires hundreds or thousands iterations. In each iteration, a propagation problem for a given waveguide bend must be solved. The finite-difference time-domain (FDTD) method is widely used, but one must use a small grid size to resolve interfaces with high index-contrast and a large computational domain to prevent waves reflected from the computation boundaries to contaminate the true solution. For numerical stability reasons, the time step must also be small accordingly. For 3D structures, such as the PhC slab, frequency domain numerical methods, such as the finite element method, are also prohibitively expensive. Since a full 3D optimization is extremely time consuming, the effective index method is widely used to obtain simpler 2D models. Although 2D models are not always accurate, the optimized structures from the 2D models still serve as good starting points for further consideration.

For ideal 2D PhC structures involving infinitely long dielectric rods or air-holes, the FDTD method still requires small grid sizes, small step sizes and large computational domains, but more efficient frequency domain methods exist. The multiple scattering technique based on cylindrical wave expansions and the multiple multipole method are accurate and efficient computational methods, since they take advantage of the available analytic solutions and the simple circular geometry of the cylinders. These methods have been successfully used to design waveguide bends in [8, 17]. Accurate solutions can also be obtained by the general-purpose finite element method [10].

In a recent work [24], we developed an efficient method for analyzing PhC devices based on the so-called Dirichlet-to-Neumann (DtN) maps of the unit cells. The DtN map of a unit cell is a relation between the wave field and its normal derivative given on the cell boundary. Based on the DtN maps, we can analyze a PhC device by solving the wave field on the edges of the unit cells only. A rigorous boundary condition was also developed for terminating semi-infinite PhC waveguides [24]. It is derived from splitting the wave field in a PhC waveguide into outgoing and incoming components and expanding these two components in Bloch modes. The computation of the Bloch modes is also greatly simplified by the DtN maps. The boundary condition allows us to use very small computational domains without loss of accuracy. The method was validated by many numerical examples involving comparisons with the FDTD, finite element and multiple multipole methods [24].

To maximize the transmittance over a frequency range, it is necessary to select a few frequencies ω_j, $j = 1, 2, ..., p$, calculate the transmittance $T(\omega_j)$ for each frequency and formulate an optimization problem. Typically, we choose $3 \le p \le 5$. Some possible formulations are

$$\max \sum_{j=1}^{p} T(\omega_j), \quad \min \sum_{j=1}^{p} [1 - T(\omega_j)]^2, \quad \max \min_{1 \le j \le p} T(\omega_j).$$

A global optimization scheme, such as the genetic algorithm, can be used [10, 17]. However, our results are obtained based on the popular BFGS quasi-Newton method [25] which is a local optimization method and it is available in MATLAB as the function `fminunc`.

3 60° bends

We consider 60° waveguide bends in an ideal 2D PhC formed by a triangular lattice of parallel and infinitely long air-holes in a dielectric medium with refractive index $n_e = 2.76$. The radius of the air-holes is $0.29a$, where a is the lattice constant. A waveguide is formed by filling a row of air-holes. For the H polarization, this structure was used to model a PhC slab where the slab thickness is $0.6a$, the refractive index is 3.4, and the radius of the air-holes is $0.29a$ [7, 26]. In such a hole-type PhC, waveguide bends with high transmission over a large frequency interval are more difficult to achieve. The standard

60° bend is shown in Fig. 1(a). A longer bend is shown in Fig. 1(b) and it is obtained by moving the hole at the center of the bend to the other side. The transmission spectra of these two bends are shown in Fig. 2 (top panel). For the standard 60° bend, our result (T_a in Fig. 2) agrees only qualitatively with the FDTD result shown as Fig. 4(b) of [26]. In particular, we obtain a maximum transmission at $\omega a/(2\pi c) \approx 0.274$, but the result in [26] gives $\omega a/(2\pi c) \approx 0.271$. Notice that we have previously validated our method for computing transmission spectra of waveguide bends. Excellent agreement was obtained for a 60° waveguide bend with a FDTD result given in [27]. Danaie *et al.* [9] considered waveguide bends in a slightly different 2D PhC where the radius of the holes is $0.3a$. We also calculated the transmission spectrum for a standard 60° bend in such a 2D PhC, and obtained good agreement with their FDTD result.

To improve the performance of the 60° bends, Chutinan *et al.* [7] proposed two bends with three or five additional air-holes of radius $0.14a$ in the waveguide core. The structures are shown in Fig. 1(c) and Fig. 1(d), respectively. Using the DtN map method developed in [24], we calculate the transmission spectra for these two bends. Our results are shown in Fig. 2 as T_c and T_d (the dash-dot curves in the middle and low panels). For the bend with three additional holes, both our result and the FDTD result (shown as Fig. 3(a) in [7]) reveal the same feature that the transmittance decreases with increasing frequency in this frequency range. However, the transmittance we obtained is about 9% higher than the one given in [7]. On the other hand, for the bend with five additional air-holes, the agreement between our result and the FDTD result (shown as Fig. 3(b) in [7]) is quite good.

We consider further optimizations of the 60° bend in the frequency interval $0.269 \leq \omega a/(2\pi c) \leq 0.282$. As discussed in [26] and [7], the 2D problem is considered as a model for a PhC slab, therefore it is only necessary to optimize the structure for a frequency interval where the waveguide in the original PhC slab has a single propagating mode. Chutinan *et al.* [26] identified the frequency range $0.27 \leq \omega a/(2\pi c) \leq 0.28$ for single mode operation of the waveguide in the PhC slab. We choose the consider a slightly larger frequency interval for the 2D model.

First, we consider the case of adding three possibly different air-holes in the bend. To maintain the symmetry between the input and output waveguides, the two non-central air-holes are assumed to be identical. After solving an optimization problem with two degrees of freedom (the radii of the two different air-holes), we obtain the structure shown in Fig. 1(e). The radii of the smaller and larger air-holes (marked in red) are $0.0598a$ and $0.1511a$, respectively. The transmission spectrum of this bend is shown as curve T_e in Fig. 2 (middle panel). Notice that the transmittance is more than 98.7% for the entire frequency interval shown in the figure. Since the radius of the smaller air-hole is quite small, we consider an alternative structure which has only two identical air-holes added

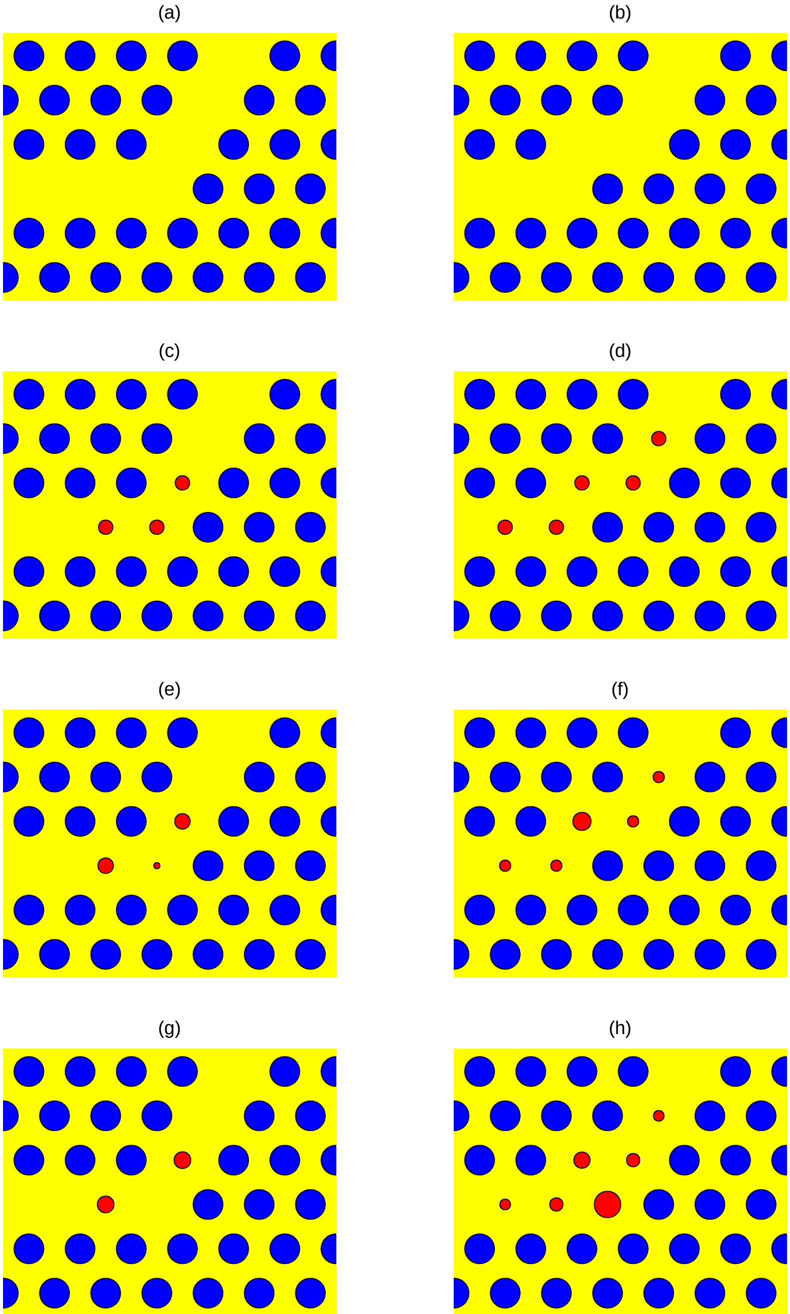

Figure 1: Waveguide bends in a 2D photonic crystal with a triangular lattice of air-holes. (a) The standard 60° bend; (b) The longer bend; (c,d) Improved bends by Chutinan *et al.* [7]; (e-h) Optimized 60° waveguide bends.

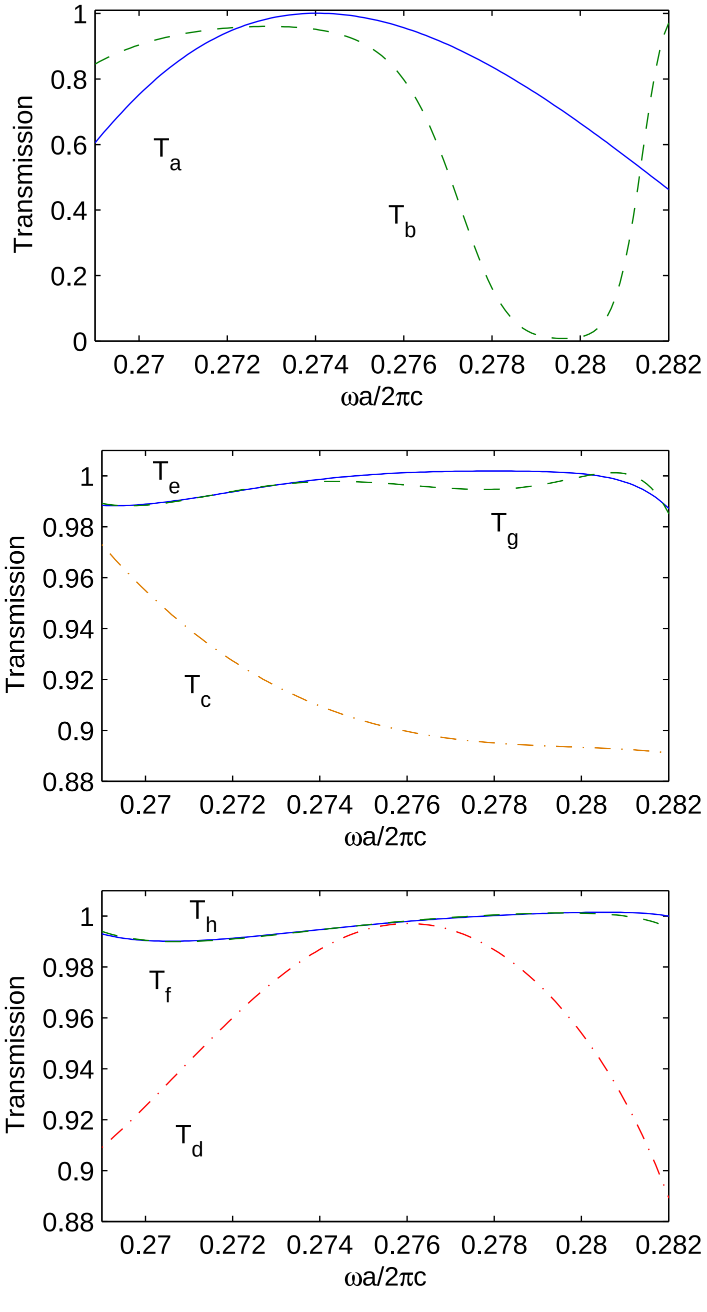

Figure 2: Transmission spectra of the 60° waveguide bends shown in Fig. 1.

to the waveguide core (without the hole at the center). Solving a simpler optimization problem with only one degree of freedom, we obtain the bend shown in Fig. 1(g), where the radius of the two air-holes is $0.1627a$. The transmission spectrum of the two-hole bend is shown as T_g in Fig. 2 (middle panel). Notice that the transmittance is more than 98.5% for the entire frequency interval. Compared with the bend with three identical holes, these two new bends show a significantly improved performance.

Next, we consider a bend with five additional air-holes in the core, but the radius of the hole at the center is allowed to be different from the other four. This leads to an optimization problem with two degrees of freedom. The resulting bend is shown in Fig. 1(f), where the radii of the larger and smaller holes are $0.1775a$ and $0.1093a$, respectively. The corresponding transmission spectrum is given as T_f in Fig. 2 (low panel). The minimum transmittance is about 99%. Finally, we consider a bend with five possibly different additional air-holes and also allow one original air-hole at the corner to change its radius. For a symmetric bend, these six air-holes can only have four different sizes. Solving an optimization problem with four degrees of freedom, we obtain the bend shown in Fig. 1(h). For the six air-holes shown in red, if we order them from lower to upper rows, and from left to right in each row, their radii are $0.1037a$, $0.1295a$, $0.2579a$, $0.1580a$, $0.1295a$ and $0.1037a$, respectively. The transmission spectrum of this bend is showed as T_h in Fig. 2 (lower panel). The minimum transmittance of this bend is also 99%. Although these two bends give slightly higher minimal transmittance, they both involve small holes with a radius about $0.1a$. In comparison, the two-hole bend shown in Fig. 1(e) is a lot easier to fabricate, since the radius of the holes is much larger ($0.1627a$).

In a recent work, Danaie *et al.* [9] considered putting five additional air-holes in the standard bend (Fig. 1(a)) instead of the longer bend (Fig. 1(b)), and solved an optimization problem with two parameters (the radii of the hole at the center and the other four holes). They used a background 2D PhC where the radius of the holes is $0.3a$ and the refractive index is 2.76. Their optimized bend involves small holes of radius $0.11a$. They achieved a transmittance of more than 90% in a wider frequency interval. In our case, the frequency interval is smaller, but the transmittance is much higher.

4 90° bends

In this section, we consider the optimization of a 90° bend. Mekis *et al.* [2] proposed a 90° bend in a 2D PhC consisting of dielectric rods on a square lattice with lattice constant a, where the medium surrounding the rods is air. The refractive index and the radius of the rods are 3.4 and $0.18a$, respectively. For the E polarization, the bulk PhC has a bandgap given by $0.302 < \omega a/(2\pi c) < 0.443$. A straight PhC waveguide is formed by removing one row of rods. The waveguide supports one propagating mode for

$0.312 < \omega a/(2\pi c) < 0.443$. The bend proposed in [2] is similar to the structure shown in Fig. 3, except that all rods should have the same radius $0.18a$. The transmission property

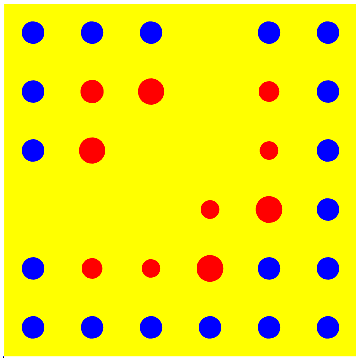

Figure 3: Optimized 90° waveguide bend in a 2D photonic crystal of dielectric rods. The radii of ten rods (shown in red) are modified.

of this bend has been analyzed by a number authors using the FDTD method [2], the finite element method [28], the multiple multipole method [8] and the DtN map method [24]. The transmission spectrum is shown in Fig. 4 as the dashed line T_1. The 90° bend has

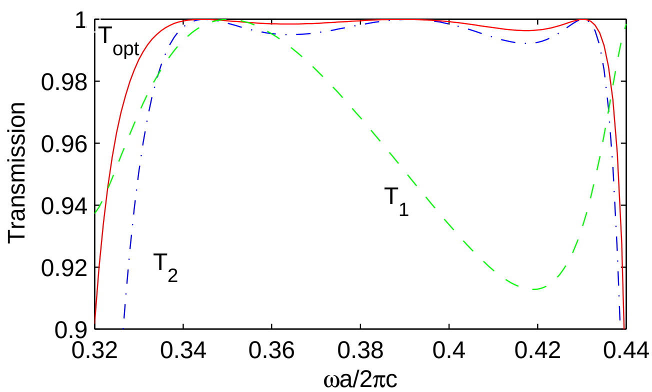

Figure 4: Transmission spectra of three different 90° photonic crystal waveguide bends. T_1, T_2 and T_{opt} correspond to the bends in [2], [8] and in Fig. 3, respectively.

been previously optimized by Smajic *et al.* [8] by varying the radii of two groups of identical rods near the center of the bend. The first group consists of three rods in the upper-left side of the bend, and the second group consists of seven rods in the lower-right side of the bend. These ten rods correspond to the red rods shown in Fig. 3, except that the rods in each group are identical. The transmission spectrum of their optimized bend is shown in Fig. 4 as the dash-dot line T_2.

We consider a further optimization of the 90° bend by allowing the ten rods (the red ones in Fig. 3) to have arbitrary radii. However, to maintain the symmetry between the input and out waveguides, we keep the structure symmetric, so that the ten rods include four identical pairs. Therefore, our optimization problem involves six degrees of freedom, namely, the radii of six distinct rods. We obtain the structure shown in Fig. 3, where the radii of six distinct rods (ordered by the columns from left to right, and in each column, from bottom to top, without repeating the identical ones) are $0.1696a$, $0.2170a$, $0.1864a$, $0.1508a$, $0.2154a$ and $0.1413a$. The transmission spectrum of this structure is shown in Fig. 4 as the solid line T_{opt}. It can be seen that more than 99.6% of the incident power can transmit through the bend in a very large frequency interval. In comparison, the bend of Smajic *et al.* [8] has a minimum power transmission around 99% in a slightly smaller frequency interval. Since their bend already has a very good performance, the room for improvement is limited.

5 Conclusion

In conclusion, based on the DtN map method developed in [24], we optimized 60° and 90° waveguide bends in 2D PhCs by varying the radii of a few dielectric rods or air-holes (including additional ones in the waveguide core). For the 60° bends, we obtained four designs involving 2, 3, 5 or 6 air-holes which are different from the regular air-holes of the PhC. These 60° bends significantly outperform the previous designs involving 3 or 5 identical additional air-holes [7]. In particular, the two-hole bend is particularly simple to fabricate, since the radius of the holes is relatively large. Although these bends are designed for ideal 2D PhCs which are invariant in the direction perpendicular to the plane of periodicity, they may serve as good initial guesses for optimizing waveguide bends in PhC slabs.

Acknowledgement

This research was partially supported by the Fundamental Research Funds for the Central Universities of China (Project No. 2009B05514), the Natural Science Funds of Hohai University (Project No. 2084/409276), and the Research Grants Council of Hong Kong Special Administrative Region, China (Project No. CityU 102207).

References

[1] J. D. Joannopoulos, R. D. Meade and J. N. Winn, Photonic Crystals: Molding the Flow of Light, Princeton University Press, Princeton, NJ. 1995.

[2] A. Mekis, J. C. Chen, I. Kurland, S. H. Fan, P. R. Villeneuve and J. D. Joannopoulos, "High transmission through sharp bends in photonic crystal waveguides," *Phys. Rev. Lett.*, vol. 77, pp. 3787-3790, 1996.

[3] T. Baba, N. Fukaya and J. Yonekura, "Observation of light propagation in photonic crystal optical waveguides with bends," *Electronics Letters*, vol. 35, pp. 654-655, 1999.

[4] M. Loncar, D. Nedeljkovic, T. Doll, J. Vuckovic, and A. Scherer, "Waveguiding in planar photonic crystals," *Applied Physics Letters*, vol. 77, pp. 1937-1939, 2000.

[5] E. Chow, S. Y. Lin, J. R. Wendt, S. G. Johnson, and J. D. Joannopoulos, "Quantitative analysis of bending efficiency in photonic-crystal waveguide bends at $\lambda = 1.55$ μm wavelengths," *Optics Letters,* vol. 26, pp. 286-288, 2001.

[6] S. Olivier, H. Benisty, M. Rattier, C. Weisbuch, M. Qiu, A. Karlsson, C. J. M. Smith, R. Houdré, and U. Oesterle, "Resonant and nonresonant transmission through waveguide bends in a planar photonic crystal," *Applied Physics Letters,* vol. 79, pp. 2514-2516, 2001.

[7] A. Chutinan, M. Okano and S. Noda, "Wider bandwidth with high transmission through waveguide bends in two-dimensional photonic crystal slabs," *Appl. Phys. Lett.*, vol. 80, pp. 1698-1700, 2002.

[8] J. Smajic, C. Hafner and D. Erni, "Design and optimization of an achromatic photonic crystal bend," *Opt. Express*, vol. 11, pp. 1378-1384, 2003.

[9] M. Danaie, A. R. Attari, M. M. Mirsalehi, and S. Naseh, "Design of a high efficiency wide-band 60° bend for TE polarization," *Photonics and Nanostructures — Fundamentals and Applications*, vol. 6, pp. 188-193, 2008.

[10] L. Rosa, K. Saitoh, K. Kakihara, and M. Koshiba, "Genetic-algorithm assisted design of C-band CROW-miniaturized PCW interleaver," *J. Lightw. Technol.*, vol. 27, pp. 2678-2687, 2009.

[11] A. Talneau, L. Le Gouezigou, N. Bouadma, M. Kafesaki, C. M. Soukoulis, and M. Agio, "Photonic-crystal ultrashort bends with improved transmission and low reflection at 1.55 μm," *Applied Physics Letters,* vol. 80, pp. 547-549, 2002.

[12] P. Strasser, G. Stark, F. Robin, D. Erni, K. Rauscher, R.Wüest, and H. Jäckel, "Optimization of a 60° waveguide bend in InP-based 2D planar photonic crystals," *J. Opt. Soc. Am. B*, vol. 25, pp. 67-73, 2008.

[13] M. F. Lu, Y. L. Yang, and Y. T. Huang, "Numerical study of transmission improvement in a photonic crystal waveguide bend by mode-matching technique," *IEEE Photonics Technology Letters,* vol. 20, pp. 2114-2116, 2008.

[14] P. I. Borel, L. H. Frandsen, A. Harpoth, J. B. Leon, H. Liu, M. Kristensen, W. Bogaerts, P. Dumon, R. Baets, V. Wiaux, J. Wouters, and S. Beckx, "Bandwidth engineering of photonic crystal waveguide bends," *Electronics Letters,* vol. 40, pp. 1263-1265, 2004.

[15] P. F. Xing, P. I. Borel, L. H. Frandsen, A. Harpøth and M. Kristensen, "Optimization of bandwidth in 60° photonic crystal waveguide bends," *Opt. Commun.*, vol. 248, pp. 179-184, 2005.

[16] G. Ren, W. Zheng, Y. Zhang, K. Wang, X. Du, M. Xing, and L. Chen, "Mode analysis and design of a low-loss photonic crystal 60° waveguide bend," *J. Lightw. Technol.*, vol. 26, pp. 2215-2218, 2008.

[17] P. Bettini, S. Boscolo, R. Specogna, and M. Midrio, "Design optimization of waveguide bends in photonic crystals," *IEEE Transactions on Magnetics*, vol. 45, pp. 1630-1633, 2009.

[18] L. H. Frandsen, A. Harpøth, P. L. Borel and M. Kristensen, "Broadband photonic crystal waveguide 60° bend obtained utilizing topology optimization," *Opt. Express*, vol. 12, pp. 5916-5921, 2004.

[19] B. Miao, C. Chen, S. Shi, J. Murakowski, D. W. Prather, "High-efficiency broadband transmission through a double-60° bend in a planar photonic crystal single-line defect waveguide," *IEEE Photonics Technology Letters,* vol. 16, pp. 2469-2471, 2004.

[20] Y. Watanabe, N. Ikeda, Y. Sugimoto, Y. Takata, Y. Kitagawa, A. Mizutani, N. Ozaki, and K. Asakawa, "Topology optimization of waveguide bends with wide, flat bandwidth in air-bridge-type photonic crystal slabs," *Journal of Applied Physics,* vol. 101, 113108, 2007.

[21] A. V. Lavrinenko, A. Tetu, H. Frandsen, J. Fage-Pedersen, and P. I. Borel, "Optimization of photonic crystal 60° waveguide bends for broadband and slow-light transmission," *Applied Physics B – Lasers and Optics*, vol. 87, pp. 53-56, 2007.

[22] A. Sharkawy, D. Pustai, S. Shi, D. W. Prather, "High transmission through waveguide bends by use of polycrystalline photonic-crystal structures," *Opt. Lett.* vol. 28, pp. 1197-1199, 2003.

[23] S. Xiao and M. Qiu, "Study of transmission properties for waveguide bends by use of a circular photonic crystal," *Physics Letters A*, vol. 340, pp. 474-479, 2005.

[24] Z. Hu and Y. Y. Lu, "Efficient analysis of photonic crystal devices by Dirichlet-to-Neumann maps," *Opt. Express*, vol. 16, pp. 17282-17399, 2008.

[25] J. Nocedal and S. J. Wright, Numerical Optimization (2nd ed.), Berlin, New York: Springer-Verlag, 2006.

[26] A. Chutinan and S. Noda, "Waveguides and waveguide bends in two-dimensional photonic crystal slabs," *Phys. Rev. B* , vol. 62, pp. 4488-4492, 2000.

[27] H. Ikuno and Y. Naka, "Finite-difference time-domain method applied to photonic crystals," in *Electromagnetic Theory and Appications for Photonic Crystals*, edited by K. Yasumoto, CRC Press, Taylor & Francis Group (2006).

[28] M. Koshiba, Y. Tsuji and M. Hikari, "Time-domain beam propagation method and its application to photonic crystal circuits," *J. Lightw. Technol.*, vol. 18, pp. 102-110, 2000.