Scattering from Periodic Arrays of Cylinders by Dirichlet-to-Neumann Maps

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***Abstract*— A simple and efficient numerical method is developed for computing the transmission and reflection spectra of periodic arrays of cylinders. For each unit cell containing a cylinder, only the wave field on the edges of the unit cell is computed. For multi-layered structures, a marching scheme based on a pair of operators is developed.**

***Index Terms*— Photonic crystals, transmission and reflection spectra, Dirichlet-to-Neumann map, cylindrical harmonic expansion.**

I. Introduction

Efficient numerical methods for simulation of light waves in photonic crystal devices are important in computer-aided design processes. Many different methods have already been developed for analyzing the scattering from a two-dimensional photonic crystal composed of arrays of dielectric or metallic cylinders. The plane wave expansion method [1] may have a slow convergence due to the discontinuity of the refractive index. The finite difference and finite element methods can be applied to general structures. In particular, high accuracy can be obtained by the adaptive finite element method [2]. However, these methods are not very efficient, since they require a discretization of the domain. The cylindrical harmonic expansion method [3], [4], [5], [6], [7], [8] is particularly powerful, since it avoids a discretization of the domain by expanding the solution around each cylinder as a series of special analytic solutions of the governing equation and solves these coefficients in a coupled linear system. The boundary integral equation method [9] has a similar advantage, since it only solves the wave field on surfaces of the cylinders. For multi-layered structures, the scattering matrix formalism [4] has been applied to the cylindrical harmonic expansion method. If the number of layers is large, the Floquet mode technique [6], [7] can be used.

Due to the periodicity of the structure along each array, sophisticated lattice sums techniques [7], [10] are needed in the cylindrical harmonic expansion method. In this paper, we develop a simple and efficient method for computing the scattering from a two-dimensional photonic crystal. Our method relies on the cylindrical harmonic expansion to compute an operator, namely, the Dirichlet-to-Neumann (DtN) map, that maps the solution to its normal derivative on the boundary of a square (or rectangle) surrounding each cylinder. The DtN map is approximated by a matrix, the size of which is identical to the number of terms retained in the cylindrical harmonic expansion. For multi-layered structures, we develop a marching scheme based on a pair of operators. The efficiency and accuracy of our method are demonstrated in a number of numerical examples.

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II. Dirichlet-to-Neumann map of a unit cell

For a two-dimensional structure specified by a z-independent refractive index function $n = n(x, y)$ and for E polarized waves propagating in the xy-plane, we have the following Helmholtz equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k_0^2 n^2(x, y) u = 0, \tag{1}$$

where u is the z-component of the electric field (the only non-zero component). For the special case where a circular cylinder of radius a is located at the origin in an otherwise homogeneous medium, we have $n = n_1$ for $r < a$ and $n = n_2$ for $r > a$, where (r, θ) is the polar coordinate system, n_1 and n_2 are the refractive indices of the cylinder and the surrounding medium, respectively. Then, the Helmholtz equation has the following general solution

$$u(x, y) = \sum_{j=-\infty}^{\infty} c_j \phi_j(r) e^{ij\theta}, \tag{2}$$

where ϕ_j is related to the Bessel functions J_j and Y_j as

$$\phi_j(r) = \begin{cases} J_j(k_0 n_1 r), & r < a, \\ A_j J_j(k_0 n_2 r) + B_j Y_j(k_0 n_2 r), & r > a. \end{cases} \tag{3}$$

The coefficients A_j and B_j can be solved from the following two equations derived from the continuity of ϕ and ϕ' at $r = a$:

$$\begin{aligned} J_j(k_0 n_2 a) A_j + Y_j(k_0 n_2 a) B_j &= J_j(k_0 n_1 a) \\ n_2 [J_j'(k_0 n_2 a) A_j + Y_j'(k_0 n_2 a) B_j] &= n_1 J_j'(k_0 n_1 a). \end{aligned}$$

Notice that the derivatives of the Bessel functions are given by

$$\begin{aligned} J_j'(s) &= [J_{j-1}(s) - J_{j+1}(s)]/2, \\ Y_j'(s) &= [Y_{j-1}(s) - Y_{j+1}(s)]/2. \end{aligned}$$

Given a closed curve Γ enclosing the cylinder, the Dirichlet-to-Neumann (DtN) map is the operator Λ that maps u on Γ to the normal derivative of u on Γ. That is

$$\Lambda \; u|_\Gamma = \left. \frac{\partial u}{\partial \nu} \right|_\Gamma,$$

where ν is a unit normal vector of Γ. If we sample the curve Γ by p points: (x_k, y_k) for $k = 1, 2, ..., p$, we can approximate

Λ by a $p \times p$ matrix. Let (r_k, θ_k) be the polar coordinates of (x_k, y_k) and ν_k be a unit normal vector of Γ at (x_k, y_k), we truncate (2) to p terms as

$$u = \sum_{j=-p/2}^{p/2-1} c_j \phi_j(r) e^{ij\theta} \quad \text{or} \quad u = \sum_{j=-(p-1)/2}^{(p-1)/2} c_j \phi_j(r) e^{ij\theta}$$

for an even or odd integer p, respectively. For an even integer p, at the given points on Γ, we have

$$u(x_k, y_k) = \sum_{j=-p/2}^{p/2-1} c_j \phi_j(r_k) e^{ij\theta_k}, \tag{4}$$

$$\frac{\partial u}{\partial \nu_k}(x_k, y_k) = \nu_k \cdot \sum_{j=-p/2}^{p/2-1} c_j \nabla[\phi_j(r_k) e^{ij\theta_k}] \tag{5}$$

for $k = 1, 2, ..., p$, where

$$\nabla[\phi_j(r) e^{ij\theta}] = \phi'(r) e^{ij\theta} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \frac{ij\phi(r) e^{ij\theta}}{r} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}.$$

Equations (4) and (5) can be written in the following matrix forms

$$\vec{u} = \mathcal{A}\vec{c}, \quad \vec{w} = \mathcal{B}\vec{c},$$

where

$$\vec{u} = [u(x_1, y_1), u(x_2, y_2), ..., u(x_p, y_p)]^T,$$
$$\vec{c} = \left[c_{-\frac{p}{2}}, c_{-\frac{p}{2}+1}, ..., c_{\frac{p}{2}-1}\right]^T,$$
$$\vec{w} = \left[\frac{\partial u}{\partial \nu_1}(x_1, y_1), \frac{\partial u}{\partial \nu_2}(x_2, y_2), ..., \frac{\partial u}{\partial \nu_p}(x_p, y_p)\right]^T.$$

Therefore, we obtain a matrix approximation of the DtN map $\Lambda = \mathcal{B}\mathcal{A}^{-1}$, such that

$$\vec{w} = \Lambda \vec{u}.$$

Related to two-dimensional photonic crystals composed of a square lattice, we consider the DtN map of a square unit cell. In this case, the curve Γ is the boundary of the square given by $|x| < L/2$ and $|y| < L/2$, where $L/2 > a$. For each edge of the square, we choose N points. For example, on the top edge of the square, the N points are

$$\left(-0.5L + \frac{k - 0.5}{N} L, \, 0.5L\right), \quad k = 1, 2, ..., N. \tag{6}$$

Notice that the four corners of the square are avoided. The normal derivative of Γ at these $4N$ points can be chosen as the partial derivatives with respect to x (for the two vertical edges at $x = \pm 0.5L$) or y (for the two horizontal edges at $y = \pm 0.5L$). Since $p = 4N$, the $(4N) \times (4N)$ matrix $\Lambda = \mathcal{B}\mathcal{A}^{-1}$ can be calculated in $O(N^3)$ operations.

III. Problem formulation

We are concerned with the transmission and reflection of periodic arrays of cylinders as in a two-dimensional photonic crystal. This is a special case of the diffractive optics problem. In this section, we recall the basic mathematical formulation of this problem and write down matrix approximations to the boundary conditions.

We consider the Helmholtz equation (1) in the entire xy plane, but assume that $n = n_b$ for $y < 0$ and $n = n_0$ for $y > D$. For $0 < y < D$, $n(x, y)$ is periodic in x with period L, i.e.,

$$n(x, y) = n(x + L, y), \quad 0 < y < D.$$

For $y > D$, we have an incident wave

$$u^{(i)}(x, y) = e^{i[\alpha_0 x - \beta_0 (y - D)]}$$

that propagates in decreasing y direction. Assuming that the time dependence is $e^{-i\omega t}$, we have $\beta_0 > 0$. Let the angle between the wave vector $(\alpha_0, -\beta_0)$ and the y axis be θ_0, we have

$$\alpha_0 = k_0 n_0 \sin\theta_0, \quad \beta_0 = k_0 n_0 \cos\theta_0.$$

The incident wave $u^{(i)}$ leads to a reflected wave $u^{(r)}$ and a transmitted wave $u^{(t)}$ which can be written down as

$$u^{(r)}(x, y) = \sum_{j=-\infty}^{\infty} R_j e^{i[\alpha_j x + \beta_j (y - D)]}, \quad y > D \tag{7}$$

$$u^{(t)}(x, y) = \sum_{j=-\infty}^{\infty} T_j e^{i[\alpha_j x - \gamma_j y]}, \quad y < 0, \tag{8}$$

where

$$\alpha_j = \alpha_0 + \frac{2j\pi}{L}, \quad \beta_j = \sqrt{k_0^2 n_0^2 - \alpha_j^2}, \quad \gamma_j = \sqrt{k_0^2 n_b^2 - \alpha_j^2}.$$

Meanwhile, the periodicity of the structure in x leads to

$$u(L, y) = \rho u(0, y), \quad \frac{\partial u}{\partial x}(L, y) = \rho \frac{\partial u}{\partial x}(0, y), \tag{9}$$

for $\rho = e^{i\alpha_0 L}$. The problem is to determine the transmission and reflection coefficients $\{T_j, R_j\}$.

Boundary conditions at $y = 0$ and $y = D$ can be written down as [11]

$$\frac{\partial u}{\partial y} = -i\tilde{S}_b u, \quad y = 0, \tag{10}$$

$$\frac{\partial u}{\partial y} = i\tilde{S}_0 u - 2i\beta_0 e^{i\alpha_0 x}, \quad y = D, \tag{11}$$

for two properly defined operators $\tilde{S}_0$ and $\tilde{S}_b$. For $u^{(t)}$ given in (8), we have

$$\frac{\partial u^{(t)}}{\partial y} = -i \sum_{j=-\infty}^{\infty} \gamma_j T_j e^{i(\alpha_j x - \gamma_j y)}.$$

Therefore, if we define a linear operator $\tilde{S}_b$ satisfying

$$\tilde{S}_b \, e^{i\alpha_j x} = \gamma_j e^{i\alpha_j x}, \quad j = 0, \pm 1, \pm 2, ... \tag{12}$$

then, from the principle of superposition, we have $\partial_y u^{(t)} = -i\tilde{S}_b u^{(t)}$. The boundary condition (10) is obtained, since $u = u^{(t)}$ for $y < 0$ and both u and $\partial_y u$ are continuous at $y = 0$. Similarly, the boundary condition (11) is obtained, if we define the linear operator $\tilde{S}_0$ by

$$\tilde{S}_0 \, e^{i\alpha_j x} = \beta_j e^{i\alpha_j x}, \quad j = 0, \pm 1, \pm 2, ... \tag{13}$$

Since $\alpha_j = \alpha_0 + 2\pi j/L$, the operator $\tilde{S}_b$ is related to a linear operator S_b defined on the Fourier components as

$$S_b \, e^{i2\pi jx/L} = \gamma_j \, e^{i2\pi jx/L}, \quad j = 0, \pm 1, \pm 2, ...$$

Let f be a periodic function of x with period L given by its Fourier series as

$$f(x) = \sum_{j=-\infty}^{\infty} \hat{f}_j e^{i2\pi jx/L}. \tag{14}$$

then

$$\begin{aligned}(S_b f)(x) &= \sum_{j=-\infty}^{\infty} \gamma_j \hat{f}_j e^{i2\pi jx/L}, \\ (\tilde{S}_b f)(x) &= e^{i\alpha_0 x} S_b[e^{-i\alpha_0 x} f(x)].\end{aligned}$$

Similarly,

$$\begin{aligned}(S_0 f)(x) &= \sum_{j=-\infty}^{\infty} \beta_j \hat{f}_j e^{i2\pi jx/L}, \\ (\tilde{S}_0 f)(x) &= e^{i\alpha_0 x} S_0[e^{-i\alpha_0 x} f(x)].\end{aligned}$$

If x is discretized by N points (say, x_k for $k = 1, 2, ..., N$), the operators $\tilde{S}_b$ and $\tilde{S}_0$ can be approximated by $N \times N$ matrices. The Fourier series (14) is truncated to N terms and approximated by the discrete Fourier transform:

$$f(x_k) = \sum_{j=-q}^{q} e^{i2\pi jx_k/L} \hat{f}_j, \quad k = 1, 2, ..., N,$$

where $q = (N-1)/2$ for an odd integer N. For an even N, the integer j is truncated from $-N/2$ to $N/2 - 1$. This gives rise to an $N \times N$ matrix $\mathcal{F}$, the (k, j') entry of which is $e^{i2\pi jx_k/L}$ for $j = j' - 1 - q$. Therefore, the operator S_b is approximated by the matrix

$$S_b = \mathcal{F}\mathcal{D}_\gamma \mathcal{F}^{-1},$$

where $\mathcal{D}_\gamma$ is the diagonal matrix of γ_j for $j = -q, -q+1, ..., q$. This leads to the matrix approximation:

$$\tilde{S}_b = \mathcal{D}_\alpha \mathcal{F}\mathcal{D}_\gamma \mathcal{F}^{-1} \mathcal{D}_\alpha^{-1},$$

where $\mathcal{D}_\alpha$ is the diagonal matrix of $e^{i\alpha_0 x_k}$ for $k = 1, 2, ..., N$. Similarly, the matrix approximations of S_0 and $\tilde{S}_0$ are

$$S_0 = \mathcal{F}\mathcal{D}_\beta \mathcal{F}^{-1}, \quad \tilde{S}_0 = \mathcal{D}_\alpha \mathcal{F}\mathcal{D}_\beta \mathcal{F}^{-1} \mathcal{D}_\alpha^{-1},$$

where $\mathcal{D}_\beta$ is the diagonal matrix of β_j for $j = -q, -q + 1, ..., q$.

To be consistent with the discretization (6) for computing the DtN map of the unit cell, the x variable (now for $0 < x < L$) is discretized as

$$x_k = \frac{k - 0.5}{N} L, \quad k = 1, 2, ..., N.$$

The matrices $\tilde{S}_0$ and $\tilde{S}_b$ can be easily calculated by a direct evaluation of the formulas or using the Fast Fourier Transform (FFT) in $O(N^3)$ or $O(N^2 \log_2 N)$ operations.

IV. Operator marching scheme

Many numerical methods [12], [13], [14], [15], [16], [17], [2] have been developed for the diffractive optics problem. Most of these methods are not specially designed for multi-layers of circular cylinders as in photonic crystal problems. In this section, we present a DtN operator marching method that utilizes the DtN map Λ of the unit cell to avoid the discretization of the interior of the unit cell completely. Our method relies on additional operators Q, Y and M, where Q and M are DtN maps defined in different context. These operators can be approximated by relatively small matrices.

The boundary value problem (1,9,10,11) can be reformulated as an initial value problem in y for the DtN map Q and fundamental solution (FS) operator Y defined (at each y) by

$$Q(y)\, u(x, y) = \frac{\partial u}{\partial y}(x, y), \quad Y(y)\, u(x, y) = u(x, 0),$$

where u is any solution of (1,9,10). Notice that condition (11) is removed from the above definition. The operators Q and Y satisfy some first order differential equations in y [18], [19], [20]. The initial conditions can be obtained from (10) and the definition of Y, that is:

$$Q(0) = -i\tilde{S}_b, \quad Y(0) = I, \tag{15}$$

where I is the identity operator. If $Q(D)$ and $Y(D)$ are calculated, we can find u at $y = D$ for the specific incident wave from (11). Therefore

$$[Q(D) - i\tilde{S}_0]\, u(x, D) = -2i\beta_0 e^{i\alpha_0 x}. \tag{16}$$

The reflected wave is obtained simply from

$$u^{(r)}(x, D+) = u(x, D) - u^{(i)}(x, D+). \tag{17}$$

Meanwhile, the FS operator $Y(D)$ gives

$$u(x, 0) = Y(D)\, u(x, D) \tag{18}$$

and the above is exactly the transmitted wave $u^{(t)}(x, 0-)$.

For a multi-layered structure, we have different layers separated in the y direction by y_0, y_1, ..., y_m and they satisfy

$$0 = y_0 < y_1 < ... < y_m = D.$$

In that case, we can find formulas that march Q and Y from y_j to y_{j+1}, if we first calculate the following DtN map M of the segment (y_j, y_{j+1}):

$$M \begin{bmatrix} u_j \\ u_{j+1} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} u_j \\ u_{j+1} \end{bmatrix} = \begin{bmatrix} \partial_y u_j \\ \partial_y u_{j+1} \end{bmatrix} \tag{19}$$

where $u_j = u(x, y_j)$, $\partial_y u_j = \partial_y u(x, y_j)$, etc. We can replace $\partial_y u_j$ and $\partial_y u_{j+1}$ above by $Q(y_j)u_j$ and $Q(y_{j+1})u_{j+1}$, respectively, then eliminate u_j. This gives rise to

$$Q(y_{j+1}) = M_{22} + M_{21}[Q(y_j) - M_{11}]^{-1} M_{12}. \tag{20}$$

From the first equation in (19) and the condition $Y(y_j)u_j = Y(y_{j+1})u_{j+1} = u_0$, we obtain

$$Y(y_{j+1}) = Y(y_j)[Q(y_j) - M_{11}]^{-1} M_{12}. \tag{21}$$

In our case, we first calculate the DtN map Λ of the rectangle given by $0 < x < L$ and $y_j < y < y_{j+1}$, that is

$$\Lambda \begin{bmatrix} u_j \\ v_{0j} \\ v_{1j} \\ u_{j+1} \end{bmatrix} = \begin{bmatrix} \partial_y u_j \\ \partial_x v_{0j} \\ \partial_x v_{1j} \\ \partial_y u_{j+1} \end{bmatrix}, \quad (22)$$

where

$$v_{0j}(y) = u(0, y), \; v_{1j}(y) = u(L, y), \quad y_j < y < y_{j+1}.$$

The matrix operator Λ can be naturally partitioned as 4×4 blocks. From the second and third equations in (22) and the condition (9), we can eliminate $\partial_x v_{0j}$ and solve v_{0j}. Then, we insert v_{0j} into the first and fourth equations in (22) and obtain

$$M = \begin{bmatrix} \Lambda_{11} & \Lambda_{14} \\ \Lambda_{41} & \Lambda_{44} \end{bmatrix} + \begin{bmatrix} C_1 D_1 & C_1 D_2 \\ C_2 D_1 & C_2 D_2 \end{bmatrix}, \quad (23)$$

where C_1, C_2, D_1, D_2 are operators given by

$$\begin{aligned} C_1 &= \Lambda_{12} + \rho\Lambda_{13} \\ C_2 &= \Lambda_{42} + \rho\Lambda_{43} \\ D_0 &= \rho\Lambda_{22} + \rho^2\Lambda_{23} - \Lambda_{32} - \rho\Lambda_{33} \\ D_1 &= D_0^{-1}(\Lambda_{31} - \rho\Lambda_{21}) \\ D_2 &= D_0^{-1}(\Lambda_{34} - \rho\Lambda_{24}). \end{aligned}$$

In conclusion, we start with $Q(0)$ and $Y(0)$ given in (15), then march Q and Y in the y direction by (20) and (21). Once $Q(D)$ and $Y(D)$ are obtained, we calculate the total field at $y = D$ by (16), the reflected wave by (17) and the transmitted wave by (18). In each marching step, we need the operator M which can be obtained from Λ as in (23). In the discrete case, all operators are represented by matrices. If $x \in (0, L)$ is discretized by N points, the operators Q, Y and M_{ij} all become $N \times N$ matrices. In each step, the marching formulas (20) and (21) require $O(N^3)$ operations. The DtN operators Λ and M can also be obtained in $O(N^3)$ operations. However, if the segments are identical, the repeated calculations of Λ and M can be avoided. For a structure with m layers, the total required number of operations is $O(mN^3)$.

V. Numerical examples

To validate our method, we calculate the transmission and reflection spectra for a number of examples and compare our results with those published by other authors. We first consider a square lattice of air-holes in a dielectric medium. As in [1], we let

$$n_0 = n_b = n_1 = 1, \quad n_2^2 = 2.1, \quad a = \frac{47.5}{170} L,$$

where a is the radius of the air-holes. The structure has $m = 16$ layers, so that $y_j = jL$ for $j = 0, 1, ..., m$. The air-holes are located at the center of each unit square. For a normal incident wave (thus, $\theta_0 = 0$, $\alpha_0 = 0$, $\beta_0 = k_0 n_0$ and $\rho = 1$), we obtain the transmission spectrum as in Fig. 1 with $N = 9$. The horizontal axis in Fig. 1 is the normalized frequency $\omega L/(2\pi c)$, where c is the speed of light in vacuum. The vertical axis is $|T_0|^2$, where T_0 is the transmission coefficient defined in (8). Our results are nearly identical to those in [1]

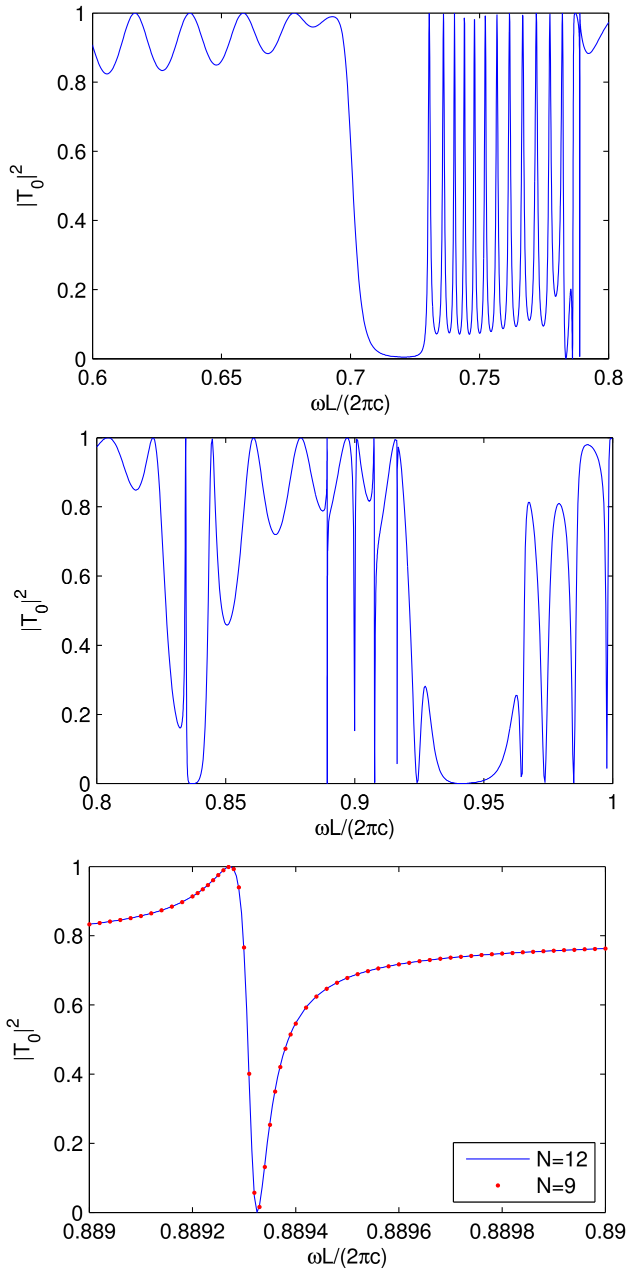

Fig. 1. Transmission spectrum of a 16 layer air-holes in a dielectric medium with refractive index $n_2 = \sqrt{2.1}$.

where a plane wave expansion of 2700 terms are used. In particular, Sakoda[1] observed some "singular" interference patterns due to simultaneous excitation of two eigenmodes (as in the bandgap calculation). In the third plot of Fig. 1, we observe that the near singular interference pattern is in fact smooth. To check the convergence, we calculate T_0 at a fixed normalized frequency $\omega L/(2\pi c) = 0.7$ for $N = 1, 2, 3, ..., 20$. Let $T_0^{(N)}$ be the T_0 calculated with N points on each edge of the unit cell, we use $T_0^{(20)}$ as a reference solution to define the relative error $E_N = |T_0^{(N)} - T_0^{(20)}|/|T_0^{(20)}|$. In Fig. 2, we plot the relative errors for $N = 1, 2, .., 16$. It is clear that the errors decrease exponentially as N is increased.

For another example, we consider the photonic crystal

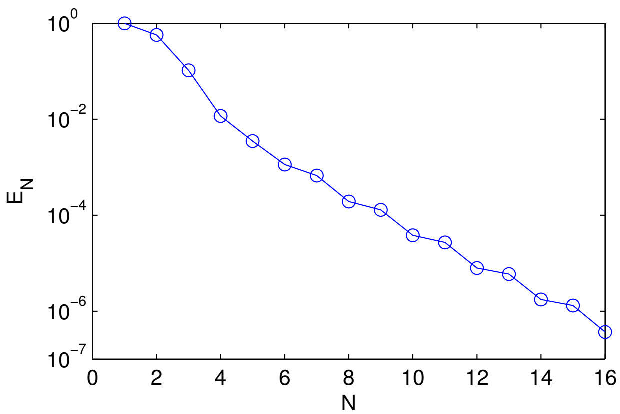

Fig. 2. Exponential decrease of the relative error of the transmission coefficient T_0 with N for $\omega L/(2\pi c) = 0.7$.

Fabry-Perot structure studied by Venakides *et al.* in [9]. There are eight layers of dielectric cylinders in free space given as a square lattice with a lattice constant L, except that the distance between the fourth and fifth layers is increased to $3.6L$. The radius and the refractive index of the cylinders are $a = 0.1524L$ and $n_1 = \sqrt{12}$, respectively. To use the techniques developed in section 3, we let

$$y_0 = 0, \ \ y_j = y_{j-1}+L \ \text{ for } \ 1 \le j \le 9, \ \ \text{except } \ y_5 = y_4+2.6L.$$

For one period $0 < x < L$, the eight cylinders are located at the center of the square unit cells given by $y_{j-1} < y < y_j$ for $1 \le j \le 9$ but $j \ne 5$. The interval $y_4 < y < y_5$ corresponds to the extra free space between the fourth and fifth layers of cylinders. The DtN map of the square unit cell containing a cylinder can be found by the procedure developed in section 2. For the rectangular region of the free space between y_4 and y_5, we can find its DtN map based on decomposing the wave field as forward and backward components in y. For a normal incident wave, we obtain the transmission spectrum of the structure as shown Fig. 3. Our results are identical to

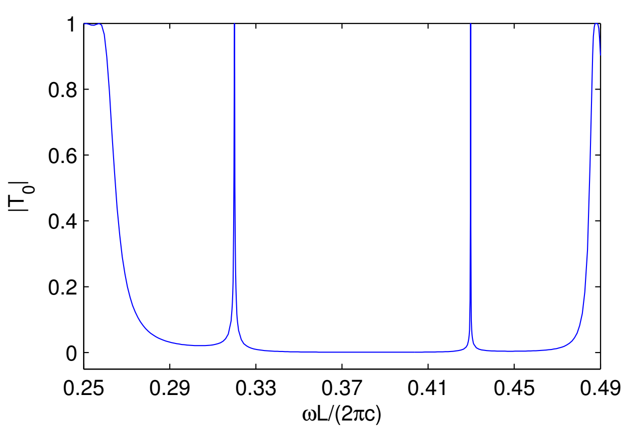

Fig. 3. Transmission coefficient of the photonic crystal Fabry-Perot structure studied in [9].

those in [9]. Venakides *et al.* developed a boundary integral equation method that solves the wave field on surfaces of the eight cylinders together.

Finally, we consider an example studied by Yasumoto *et al.* in [7]. Two layers of dielectric cylinders are given in a rectangular lattice. The distance between nearby cylinders in the x direction (which extends to infinity periodically) is L, but the distance in the y direction (between the two layers) is $0.7L$. The radius of the cylinders is $a = 0.3L$ and the refractive index of the cylinders is $n_1 = \sqrt{2}$. Our calculations with $N = 9$ confirm the results in [7]. The reflection spectrum of this two layer periodic structure is shown in Fig. 4.

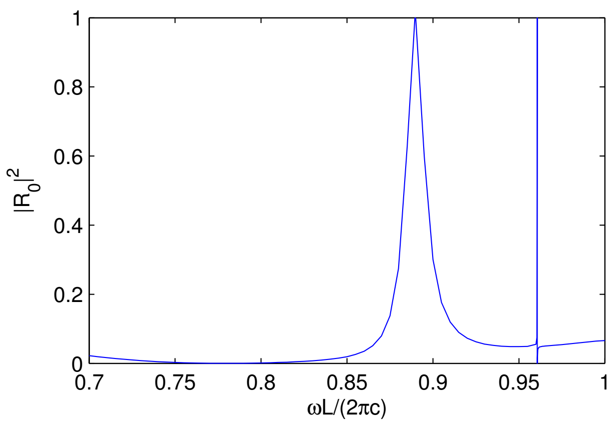

Fig. 4. Reflection spectrum of two layers of dielectric cylinders with a reduced distance between the layers.

VI. Conclusions

We have developed a simple and efficient method for computing transmission and reflection spectra of a 2-D photonic crystal structures composed of cylinders or air-holes in a medium. Our method uses the Dirichlet-to-Neumann (DtN) map Λ of a unit cell to satisfy the continuity condition on edges of the unit cells. The cylindrical harmonic expansion method is used to find a matrix approximation of Λ. For multi-layered structures, an operator marching scheme is developed based on a pair of operators. Our method avoids the lattice sums techniques needed in standard cylindrical harmonic expansion methods. Numerical results suggest that the errors decrease exponentially with N, where N is the number of collocation points on each edge of the unit cell.

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