

Bilateral symmetry breaking in nonlinear circular cylinders

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Abstract: Symmetry breaking is a common phenomenon in nonlinear systems, it refers to the existence of solutions that do not preserve the original symmetries of the underlying system. In nonlinear optics, symmetry breaking has been previously investigated in a number of systems, usually based on simplified model equations or temporal coupled mode theories. In this paper, we analyze the scattering of an incident plane wave by one or two circular cylinders with a Kerr nonlinearity, and show the existence of solutions that break a lateral reflection symmetry. Although symmetry breaking is a known phenomenon in nonlinear optics, it is the first time that this phenomenon was rigorously studied in simple systems with one or two circular cylinders.

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1. Introduction

In nonlinear systems with a certain symmetry, besides the solutions that possess the same symmetry, there could be other solutions that do not preserve that symmetry. This so-called symmetry breaking phenomenon shows up in many nonlinear systems, can be driven by different nonlinearities, and is typically associated with a pitchfork bifurcation. In nonlinear optics, symmetry breaking caused by the instantaneous Kerr nonlinearity has been investigated by a number of authors [1–15]. Akhmediev [1] found asymmetric modes in a symmetric structure consisting of a linear layer between two nonlinear layers. Some of these works on symmetry breaking are based on nonlinear resonators (cavities) illuminated by two incident waves with equal intensities and opposite directions [2, 3, 10, 11, 13, 15]. The systems in these studies have a reflection symmetry along the axis of main propagation, but solutions breaking the reflection symmetry can exist for certain range of the incident power. In other studies [4, 8, 9, 14], the incident waves are specified in one side of the resonators, but the systems have a reflection symmetry in lateral directions (perpendicular to the main propagation direction), and solutions breaking the lateral reflection symmetry can also exist. This kind of symmetry breaking was referred to as bilateral symmetry breaking [9]. Symmetry breaking has also been extensively studied for coupled nonlinear Schrödinger equations in connection with solitons in fiber couplers [5–7] and Bose-Einstein condensates [16–18].

The earlier studies on symmetry breaking [2–4, 8, 9] are based on simplified models which capture the main physics without providing accurate wave field distributions. The more recent works [10–15] are based on the temporal coupled mode theory (CMT) for photonic crystal (PhC) waveguides and cavities. When the parameters are calculated from numerical simulations for the individual elements of the structure, the CMT models are quite accurate. Maes *et al.* [10] also performed rigorous numerical simulations for a complete structure with PhC waveguides and nonlinear cavities. In [4, 8], symmetry breaking phenomena are studied for planar nonlinear resonators, where the incident waves are given in one side of the resonator, and they are either two plane waves [4] or two finite beams [8] with identical amplitudes and opposite incident angles. Torres *et al.* [9] studied a similar resonator, but the incident waves are Gaussian beams. In all these works [4, 8, 9], the nonlinear systems include both the structures (i.e., the nonlinear resonators) and the incident waves, are symmetric in lateral directions (parallel to the resonators), but solutions breaking the lateral symmetry can exist. Bulgakov and Sadreev [14] discovered a similar bilateral symmetry breaking in a PhC waveguide T-branch

coupled with two nonlinear cavities, and analyzed this phenomenon with the temporal coupled mode theory.

In this paper, we perform rigorous numerical simulations for the scattering of plane waves by one or two circular cylinders with a Kerr nonlinearity, and show that solutions breaking a bilateral reflection symmetry could exist in these systems. The circular cylinders are the simplest two-dimensional (2D) structures. To the best of our knowledge, the symmetry breaking phenomenon has never been investigated in these simple structures. We believe that it is useful to study symmetry breaking and other nonlinear phenomena in simple structures such as slabs and circular cylinders, where a rigorous and detailed analysis can be performed, and the insight gained in these studies may help in the design of more sophisticated structures for practical applications of these nonlinear effects.

2. Formulation

We consider one or two circular cylinders, shown in Fig. 1 as case I and case II respectively, and

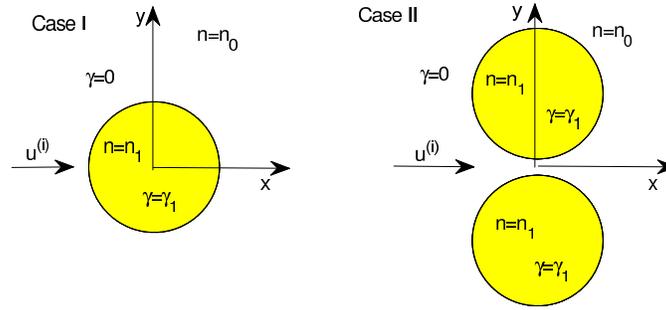


Fig. 1. One or two nonlinear cylinders parallel to the z axis and illuminated by a plane wave propagating in the x direction.

surrounded by a linear homogeneous medium of refractive index n_0 , where the cylinders are made of a nonlinear material with a third order nonlinearity. In a Cartesian coordinate system $\{x, y, z\}$, the cylinders are parallel to the z axis. For case I, the axis of the single cylinder is exactly the z axis. For case II, the cylinder axes are located at $x = 0$ and $y = \pm b$, where b is positive. The cylinders are identical, their radius and refractive index are a ($a < b$) and n_1 , respectively. The incident wave is a plane wave with amplitude A propagating in the positive x direction.

For the E polarization, the governing equation in the frequency domain is the following nonlinear Helmholtz equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k_0^2 (n^2 + \gamma |u|^2) u = 0, \quad (1)$$

where $k_0 = \omega/c$ is the free space wavenumber, ω is the angular frequency, c is the speed of light in vacuum, $n = n(x, y)$ is the refractive index function, and $\gamma = \gamma(x, y)$ is the nonlinear coefficient which vanishes in linear media. In the nonlinear cylinders, $\gamma = \gamma_1 = \frac{3}{4} \chi_{zzzz}^{(3)}$, where $\chi_{zzzz}^{(3)}$ is an element of the third order nonlinear susceptibility tensor. The unknown function $u(x, y)$ is related to the z component of the actual electric field by $E_z \approx \text{Re}(u e^{-i\omega t})$, assuming third harmonic generation can be ignored. The incident wave is the plane wave given by

$$u^{(i)}(x, y) = A e^{ik_0 n_0 x}, \quad (2)$$

where A is the amplitude. Outside the cylinders, the total field is the sum of incident and scattered waves, i.e., $u = u^{(i)} + u^{(s)}$. The scattered wave satisfies the Sommerfeld radiation condition:

$$\lim_{r \rightarrow \infty} \sqrt{r} \left[\frac{\partial u^{(s)}}{\partial r} - ik_0 n_0 u^{(s)} \right] = 0, \quad (3)$$

where $r = \sqrt{x^2 + y^2}$ is the radial variable.

Apparently, the system has a reflection symmetry in y . Therefore, solutions that are symmetric in y (i.e. even functions of y satisfying $u(x, y) = u(x, -y)$) always exist. In the following sections, we show that asymmetric solutions also exist for some range of the incident wave intensity.

3. Computational methods

Symmetry breaking and optical bistability are related to the existence of multiple solutions, i.e., non-uniqueness, of the nonlinear Helmholtz equation (1). To have full confidence in our results, we need an efficient numerical method to compute the solutions to high accuracy. Our method contains the following three aspects: (1) boundary conditions on the boundaries of the cylinders; (2) iterative schemes for the nonlinear Helmholtz equation; and (3) pseudospectral method for solving each iteration.

Instead of solving Eq. (1) on the entire xy plane, we prefer to solve it only on the nonlinear cylinders. To achieve this, we need a boundary condition on the boundaries of the cylinders. Let Ω be the cross sections of the single cylinder or the two cylinders (for case I or case II, respectively) in the xy plane, $\partial\Omega$ be the boundary of Ω , and \mathbf{v} be an outward unit normal vector of $\partial\Omega$, we show that the boundary condition can be written as

$$\frac{\partial u}{\partial \mathbf{v}} = \Lambda u + Af \quad \text{on} \quad \partial\Omega, \quad (4)$$

where f is a function defined on $\partial\Omega$ and Λ is an operator that acts on functions defined on $\partial\Omega$.

For the first case, Ω is the disk given by $r < a$, $\partial\Omega$ is the circle $r = a$, and the normal derivative on $\partial\Omega$ is simply the partial derivative with respect to r . Outside the cylinder, the scattered wave can be expanded in cylindrical waves:

$$u^{(s)}(x, y) = \sum_{m=-\infty}^{\infty} c_m H_m^{(1)}(k_0 n_0 r) e^{im\theta}, \quad r > a, \quad (5)$$

where r and θ are the polar coordinates, $H_m^{(1)}$ is the m th order Hankel function of the first kind. Since $\partial_r u^{(s)} = k_0 n_0 \sum c_m H_m^{(1)'}(k_0 n_0 r) e^{im\theta}$ where $H_m^{(1)'}$ is the derivative of $H_m^{(1)}$, if we define a linear operator Λ such that

$$\Lambda e^{im\theta} = \lambda_m e^{im\theta}, \quad \lambda_m = k_0 n_0 \frac{H_m^{(1)'}(k_0 n_0 a)}{H_m^{(1)}(k_0 n_0 a)}, \quad (6)$$

for all integer m , then $\partial_r u^{(s)} = \Lambda u^{(s)}$ at $r = a$. This leads to boundary condition (4), where $Af = \partial_r u^{(i)} - \Lambda u^{(i)}$ for $r = a$, and it can be evaluated using the Jacobi expansion of the plane incident wave. If $\partial\Omega$ (or θ) is discretized by M points, the expansion (5) should be truncated to M terms, then Λ can be approximated by an $M \times M$ matrix and it can be calculated using the discrete Fourier transform.

For case II, where Ω is the union of two disks and $\partial\Omega$ is the union of two circles, we need to use the local polar coordinates $\{r^{(1)}, \theta^{(1)}\}$ and $\{r^{(2)}, \theta^{(2)}\}$ with respect to the cylinder centers

$(0, b)$ and $(0, -b)$, respectively, then the normal derivative $\partial_\nu u$ on $\partial\Omega$ becomes $\partial u/\partial r^{(1)}$ or $\partial u/\partial r^{(2)}$ on the first or second circles, respectively. For two cylinders, the operator Λ does not have an analytic formula as (6), but it can still be approximated numerically. We note that the scattered wave outside the two cylinders can be written as

$$u^{(s)}(x, y) = \sum_{l=1}^2 \sum_{m=-\infty}^{\infty} c_m^{(l)} H_m^{(1)}(k_0 n_0 r^{(l)}) e^{im\theta^{(l)}}, \quad (7)$$

where $c_m^{(l)}$ ($l = 1, 2$) are unknown coefficients. If we discretize each circle (i.e., $\theta^{(1)}$ and $\theta^{(2)}$) by M points, and truncate the sum of m in (7) to M terms, then we can express $u^{(s)}$ at these $2M$ points on $\partial\Omega$ in terms of the $2M$ coefficients $c_m^{(l)}$. We can also express $\partial_\nu u^{(s)}$ at the $2M$ points on $\partial\Omega$ in terms of $c_m^{(l)}$. Eliminating these unknown coefficients, we obtain a $(2M) \times (2M)$ matrix such that $\partial_\nu u^{(s)} = \Lambda u^{(s)}$ on $\partial\Omega$ approximately, where $u^{(s)}$ and $\partial_\nu u^{(s)}$ have been replaced by column vectors of length $2M$. The function f is also replaced by a column vector of length $2M$, and it can be obtained by evaluating $Af = \partial_\nu u^{(i)} - \Lambda u^{(i)}$ at these $2M$ points on $\partial\Omega$.

In general, if the nonlinearity is contained in a compact domain Ω , it is advantageous to use the boundary condition (4). This condition reduces the original problem formulated on the entire xy plane to a smaller problem formulated on Ω . Furthermore, for nonlinear problems, it is usually necessary to consider many different values of A , and even for a fixed A , an iterative method is needed. The boundary condition (4) solves the linear part of the problem (outside Ω) once and for all. Notice that Λ is independent of the incident wave, and f is independent of the amplitude A . In a previous work [19], we applied this technique to a nonlinear problem in a photonic crystal waveguide-cavity system. Unlike the simpler cases considered here, there is no simple analytic expression for the wave field outside Ω , but Λ and f can still be calculated numerically. For photonic crystal structures, the method based on the Dirichlet-to-Neumann (DtN) maps of the unit cells is particularly efficient [20, 21], and it has been applied to find Λ and f in [19].

To solve Eq. (1) in Ω with the boundary condition (4), it is necessary to use an iterative method. The simplest method may be the fixed point iteration

$$\mathcal{L}u^{(j+1)} + k_0^2 \gamma |u^{(j)}|^2 u^{(j+1)} = 0, \quad (8)$$

where $u^{(j)}$ is the current iteration, $u^{(j+1)}$ is the next iteration to be solved, and $\mathcal{L} = \partial_x^2 + \partial_y^2 + k_0^2 n^2$. However, the method converges slowly and often fails to converge. Newton's method is widely used to solve various nonlinear problems. For Eq. (1), it is given as

$$\mathcal{L}u^{(j+1)} + k_0^2 \gamma \left\{ 2|u^{(j)}|^2 u^{(j+1)} + [u^{(j)}]^2 \bar{u}^{(j+1)} \right\} = 2k_0^2 \gamma |u^{(j)}|^2 u^{(j)}. \quad (9)$$

Notice that $\bar{u}^{(j+1)}$, the complex conjugate of the unknown function $u^{(j+1)}$, also appears in Eq. (9). Therefore, it is impossible to solve $u^{(j+1)}$ as a single complex function. Instead, it is necessary to rewrite Eq. (9) as a system of two equations for the real and imaginary parts of $u^{(j+1)}$, and solve these two real functions. Newton's method converges very fast if there is a good initial guess, but the convergence domain is quite small. If the initial guess is not good, Newton's method will fail to converge. One way to enlarge the convergence domain is to use the so-called quasi-Newton's method. That method has a slower convergence rate, and it also contains a parameter which is not easy to choose.

If we replace the unknown function $\bar{u}^{(j+1)}$ by the current iteration $\bar{u}^{(j)}$ in Eq. (9), the following iterative scheme is obtained:

$$\mathcal{L}u^{(j+1)} + 2k_0^2 \gamma |u^{(j)}|^2 u^{(j+1)} = k_0^2 \gamma |u^{(j)}|^2 u^{(j)}. \quad (10)$$

This method is easier to implement than Newton's method, since $u^{(j+1)}$ can be solved as a single complex function. It appears to be quite robust, that is, it converges with zero initial guess for a large range of incident power. Compared with Newton's method, it has a slower convergence rate, but we can use this method to find a good approximate solution, and further improve it by Newton's method.

The inhomogeneous Eqs. (9) and (10) can be solved by a mixed Fourier-Chebyshev pseudospectral method, where the radial and angle directions are discretized by the Chebyshev and Fourier collocation methods, respectively. A clear description of the method can be found in the book by Trefethen [22]. We have previously used this method to analyze second harmonic generation [23] and optical bistability [19] in photonic crystal structures. For the single-cylinder case, r and θ are discretized by N and M points respectively, as

$$r_p = a \cos\left(\frac{p\pi}{2N-1}\right), \quad \theta_q = \frac{\pi(2q-1)}{M}, \quad (11)$$

for $0 \leq p < N$ and $1 \leq q \leq M$, where M is an even integer. Discretizing the differential Eq. (9) or (10) and the boundary condition (4), a linear system is obtained for $u^{(j+1)}$ at all r_p and θ_q . For the case of two cylinders, we discretize the local polar coordinates $\{r^{(1)}, \theta^{(1)}\}$ and $\{r^{(2)}, \theta^{(2)}\}$ as in Eq. (11), then the final linear system involves $2NM$ unknowns. For symmetric solutions (even functions of y), the size of the linear system can be reduced by one half.

4. Case I: one cylinder

In this section, we consider a single nonlinear circular cylinder with radius $a = 0.4L$, refractive index $n_1 = 2.5$, and nonlinear coefficient $\gamma_1 = 2 \times 10^{-12} \text{ m}^2/\text{V}^2$, where $L = 1 \mu\text{m}$. The cylinder is surrounded by air ($n_0 = 1$), its axis coincides with the z axis, and it is illuminated by the incident wave given in Eq. (2). Since γ_1 is extremely small, nonlinear effects are typically very weak, unless the incident power is very large. The nonlinear effects are more evident if the frequency is near a resonant frequency, since the electromagnetic field in the structure is amplified at the resonant frequencies. As a linear structure, the circular cylinder can be regarded as a resonator and the resonant modes can be solved analytically. The resonant modes are complex-frequency solutions of the homogeneous Maxwell's equations (or Helmholtz equations for 2D structures) satisfying outgoing radiation conditions. For the given parameters a , n_1 and n_0 , the complex frequency of one resonant mode is $\omega_* L / (2\pi c) = 0.9779 - 0.0087i$. This mode is chosen, since it has a relatively large quality factor ($Q = -2\text{Re}(\omega_*) / \text{Im}(\omega_*) \approx 229.4$). However, the nonlinear effects are strongest when the real ω is slightly less than the real part of ω_* , since the effective index of the nonlinear medium is increased when $\gamma_1 > 0$. In the following, we consider the problem for $\omega L / (2\pi c) = 0.95238$.

In Fig. 2(a), we show the normalized scattered power P versus the intensity of the incident plane wave I , where P is the scattered power divided by the diameter of the cylinder $2a$, and I_0 is the intensity of a reference incident wave with amplitude $A_0 = \sqrt{10} \times 10^4 \text{ V/m}$. The blue and red curves represent the symmetric and asymmetric solutions, respectively. Since the red curve nearly overlaps with the blue curve, we show the difference in the scattered powers of the symmetric and asymmetric solutions in Fig. 2(b). From Fig. 2(a), it can be seen that the symmetric solutions exhibit a classical optical bistability phenomenon with a hysteresis loop for $20.9 \leq I/I_0 \leq 33.1$. The asymmetry solutions appear in pairs. If one solution is $u(x, y)$, then the other solution is $u(x, -y)$. They exist for $31.5 < I/I_0 < 332$ and have the same scattered power. Therefore, the red curve in Fig. 2(a) actually represents two solutions. Notice that there are five solutions (three symmetric and two non-symmetric) for $31.5 < I/I_0 < 33.1$. We expect that some of these solutions are unstable, but a stability analysis cannot be performed using Eq. (1), instead, it must start from the time-domain nonlinear Maxwell's equations. To provide

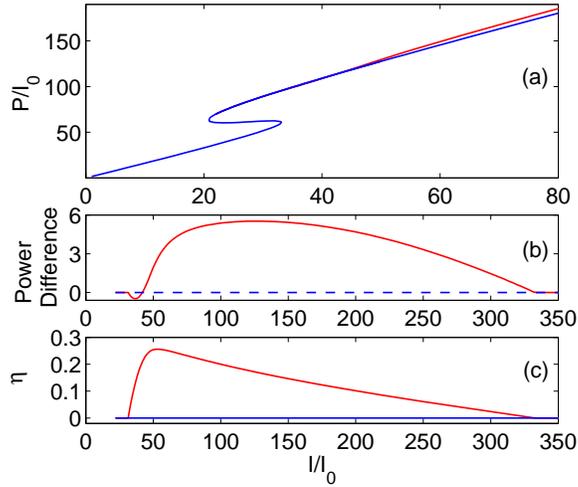


Fig. 2. Case I: (a) normalized scattered power P vs. the intensity of the incident wave I , where the blue and red curves represent symmetric and asymmetric solutions, respectively; (b) difference between the normalized scattered powers of the asymmetric and symmetric solutions; (c) the asymmetry measure η for the asymmetric solutions.

a measure of asymmetry, we define

$$\eta = \frac{\int_{\Omega} |u(x, y) - u(x, -y)|^2 dx dy}{4 \int_{\Omega} |u(x, y)|^2 dx dy} \quad (12)$$

for the asymmetric solutions, where Ω is the cross section of the cylinder, i.e., the disk $r < a$. In Fig. 2(c), we show η as a function of the incident wave intensity I .

Next, we show the electric field distributions for solutions excited by an incident wave with the intensity $I = 40I_0$. In Fig. 3(a) and (b), we show the real and imaginary parts of the symmetric solution. Those of an asymmetric solution are shown in Fig. 3(c) and (d). Since the asymmetric solutions come in pairs, there is another asymmetric solution which is just the mirror image (with respect to the x axis) of the one shown in Fig. 3.

5. Case II: two cylinders

The phenomenon of bilateral symmetry breaking also appears for a pair of nonlinear circular cylinders. We consider two identical cylinders with the same parameters as in the previous section: $n_1 = 2.5$, $\gamma_1 = 2 \times 10^{-12} \text{ m}^2/\text{V}^2$, $a = 0.4L$ and $L = 1 \mu\text{m}$. These cylinders are centered at $(0, \pm b)$ for $b = 0.45L$, surrounded by air ($n_0 = 1$) and illuminated by the plane incident wave given in Eq. (2). As a linear resonator, the cylinder pair has an infinite sequence of resonant modes. One mode with a relatively large Q factor has a complex frequency $\omega_* L / (2\pi c) = 0.9620 - 0.0063i$. We consider the nonlinear problem for the real frequency $\omega L / (2\pi c) = 0.94294$.

In Fig. 4, we show the normalized scattered power P (the scattered power divided by $2a$), the difference between the normalized scattered powers of the asymmetric and symmetric solutions, and the measure of asymmetry η , as functions of the incident wave intensity I , where η is defined in Eq. (12) with Ω being the cross sections of both cylinders. The symmetric solutions exhibit a classical optical bistability loop for $6.2 \leq I/I_0 \leq 15.7$. The asymmetric solutions

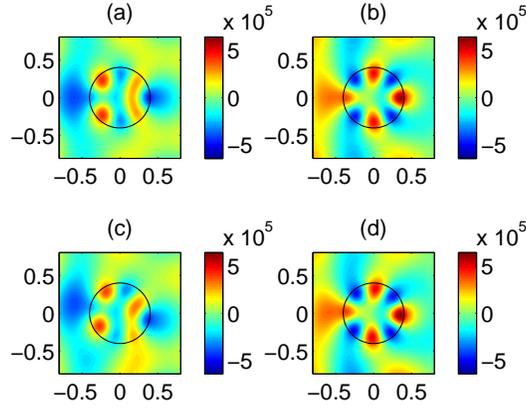


Fig. 3. Electric field distributions for case I with $I = 40I_0$. (a) and (b): real and imaginary parts of the symmetric solution; (c) and (d): real and imaginary parts of an asymmetric solution.

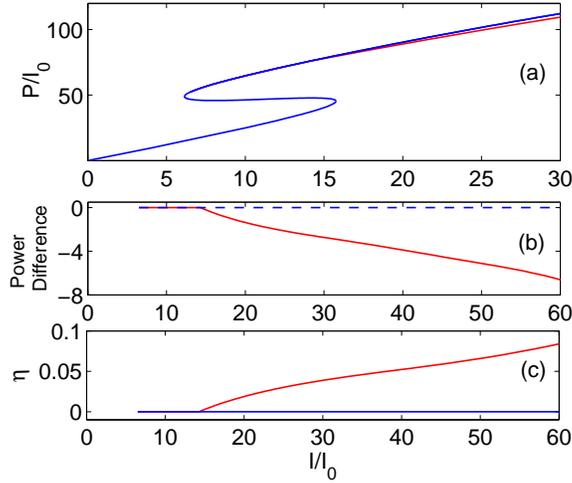


Fig. 4. Case II: (a) normalized scattered power P vs. intensity I of the incident wave, where blue and red curves represent the symmetric and asymmetric solutions; (b) difference between the normalized scattered powers of the asymmetric and symmetric solutions; (c) the measure of asymmetry η .

appear for $I/I_0 > 14.3$. In Fig. 5, we show the electric field distributions of the symmetric and asymmetric solutions for $I/I_0 = 30$. Only one asymmetric solution is shown in Fig. 5. The other asymmetric solution can be obtained by a simple reflection with respect to the x axis.

6. Conclusion

In this paper, we analyzed the scattering of plane waves by one or two circular cylinders with an instantaneous Kerr nonlinearity, and found solutions that break a reflection symmetry in the lateral direction perpendicular to the incident wave vector. Symmetry breaking can be driven by

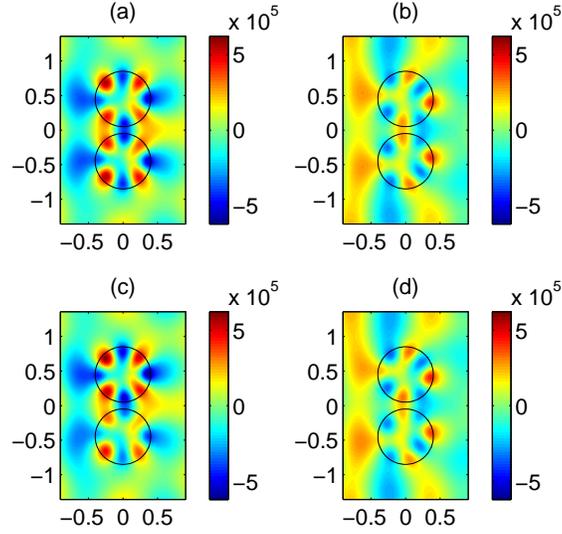


Fig. 5. Electric field distributions for case II with $I = 30I_0$. (a) and (b): real and imaginary parts of the symmetric solution; (c) and (d): real and imaginary parts of an asymmetric solution.

various nonlinearities. For the Kerr nonlinearity, symmetry breaking has been found in systems with a reflection symmetry along or perpendicular to the main propagation direction. Earlier studies on bilateral symmetry breaking [4, 8, 9] have relied on simplified models that do not provide accurate wave field distributions. The more recent study [14] uses the temporal coupled mode theory to analyze nonlinear cavities in photonic crystal structures. Our work represents a rigorous numerical study of the symmetry breaking phenomenon in simple 2D systems with circular cylinders. Since a large incident power is needed for asymmetric solutions to appear, it is difficult to make use of the symmetry breaking phenomena based on these simple systems. To realize practical applications such as optical switching, it is necessary to use more complicated systems or resonators with much higher quality factors [10–15]. Nevertheless, we believe that studies of nonlinear phenomena in simple systems are useful, since the physical insight gained can help in the design of more sophisticated systems for practical applications.

Our study is based on the nonlinear Helmholtz equation for the E polarization. Similar symmetry breaking phenomena appear for the H polarization. The study relies on an efficient numerical method that achieves high accuracy with minimal computing effort. A key step of our method is to reduce the original problem formulated on the entire xy plane to a much smaller problem on the nonlinear cylinders only. We also developed a robust iterative scheme for handling the nonlinearity and implemented a highly accurate pseudospectral method for the iterations.

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