Incentivizing Device-to-Device Load Balancing for Cellular Networks: An Online Auction Design

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***Abstract*—The device-to-device load balancing (D2D-LB) paradigm has been advocated in recent small-cell architecture design for cellular networks. The idea is to exploit inter-cell D2D communication and dynamically relay traffic of a busy cell to adjacent under-utilized cells to improve spectrum temporal efficiency, addressing a fundamental drawback of small-cell architecture. Technical challenges of D2D-LB have been studied in previous works. The potential of D2D-LB, however, cannot be fully realized without providing proper incentive mechanism for device participation. In this paper, we address this economical challenge using an online procurement auction framework. In our design, multiple sellers (devices) submit bids to participate in D2D-LB and the auctioneer (cellular service provider) evaluates all the bids and decides to purchase a subset of them to fulfill load balancing requirement with the minimum social cost. Different from similar auction design studies for cellular offloading, battery limit of relaying devices imposes a time-coupled capacity constraint that turns the underlying problem into a challenging multi-slot one. Furthermore, as the input to the problem are revealed in a slot-by-slot fashion, calling for online algorithm design for the multi-slot auction problem. We first tackle the single-slot version of the problem, show that it is NP-hard, and design a polynomial-time offline algorithm with a small approximation ratio. Building upon the single-slot results, we design an online algorithm for the multi-slot problem with sound competitive ratio. Our auction algorithm design ensures that truthful bidding is a dominant strategy for devices. Extensive experiments using real-world traces demonstrate that our proposed solution achieves near offline-optimum and reduces the cost by 45% compared to an alternative heuristic.**

***Index Terms*—Cellular networks, device-to-device load balancing, online algorithm design, approximation and competitive analysis, procurement auction design, truthful analysis.**

I. Introduction

CELLULAR traffic has witnessed an exploding growth due to advances in smartphones and content-rich applications. Global cellular traffic reached 7.8 exabytes per month in 2015, and is expected to reach about 76 exabytes per month by 2020, with a 57% annual growth rate [4]. Given the scarcity of radio spectrum for cellular communication, it has been a major challenge for cellular service providers (CSP) to serve the fast-growing traffic demand.

There are mainly two lines of efforts to tackle this challenge. The first is to offload cellular traffic to the other spectrum ranges such as WiFi or the recent 60GHz wireless band [29]. The second is to adopt a small-cell architecture [6], [17] and improve *spectrum spatial efficiency* by reducing the size of cells. This approach, however, suffers from low *spectrum temporal efficiency*. In particular, (small) cells serving a limited number of users commonly observe large temporal fluctuation in overall traffic. As CSP usually provisions spectrum to a cell according to its traffic peak, large temporal fluctuation in traffic volumes inevitably leads to low spectrum temporal efficiency. A case study [14] reported that average cell-capacity utilization is only 25% for a major CSP in a metropolitan district.

Device-to-Device Load Balancing. In inband D2D communication [16], mobile users directly communicate together using cellular spectrum. Exploiting D2D communication to balance the load between adjacent base stations, termed as *Device-to-device load balancing* (D2D-LB), has been advocated in the recent studies [14], [24] to improve spectrum temporal efficiency. The idea is motivated by the observation that traffics of adjacent cells might be uncorrelated, thereby their peaks occur at different time epochs and are asynchronous. In Fig. 1(b), an example of single-day traffics of two adjacent base stations demonstrates that their peaks are asynchronous. In Fig. 1(a), we further report the statistical correlation coefficients [9] of base stations using cell-traffic traces of 194 base stations from Smartone [3], a major CSP in Hong Kong. The results show that correlation coefficient of more than 50% of adjacent BSs is below 0.2, which is similar to the example shown in Fig. 1(b) (see Appendix A for details). Putting together above observations along with further measurements in [14], [24], we envision ample room for reducing the peak traffic by doing D2D-LB. Note that smart user association [32] can also be applied to balance cell-level traffic. This approach is complimentary to D2D-LB since it is more applicable on large timescale, whereas D2D-LB is more applicable on small timescale. We discuss the details in Sec. II.

The idea of D2D-LB is to shift a portion of the traffic of busy cells to adjacent underutilized cells using inter-cell D2D communication, as shown in Fig. 2. In particular, in this simple example, while BS_1 is heavy-loaded, its adjacent base station BS_2 is idle. By implementing D2D-LB, user u_1 transmits its

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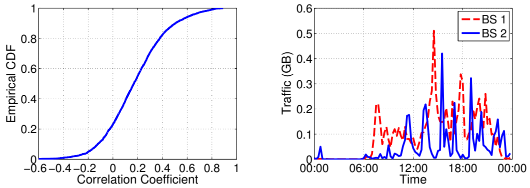


(a) (b)

Fig. 1. The empirical CDF of Pearson correlation coefficients [9] of traffics of adjacent BSs is shown in Fig. 1(a). Fig. 1(b) shows traffics in a single day of a sampled pair of adjacent BSs. See Appendix A for detailed explanations.

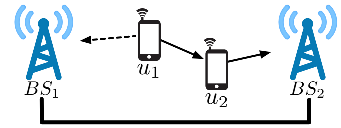


Fig. 2. BS_1 is heavy-loaded and BS_2 is idle. In D2D-LB, instead of directly transmitting data of user u_1 to BS_1 (dotted line), the data is load-balanced to BS_2 via device u_2 using the spectrum of idle cell BS_2 (solid lines).

traffic to device u_2 and then the traffic is forwarded to BS_2. Both transmissions are done using the idle spectrum of BS_2.

By implementing D2D-LB, the peak traffic of the busy cell is reduced, the free resources of the idle cells are utilized, thereby improving network-wide spectrum temporal efficiency. As a notable result, a measurement in [14] shows that D2D-LB can reduce the peak traffic of individual cells by up to 35%, which yields substantial saving of the precious spectrum resource. Note that D2D-LB and data offloading may read similar as they both shift cellular traffic around. They are, however, substantially different, which we discuss in Sec. II.

Two key challenges stand on the way towards capitalizing the spectrum-saving benefit of D2D-LB. The first is the technical challenge studied in the previous research [14], [24]. The second is the economic challenge. According to [2]: "*another perhaps bigger challenge is making people comfortable with the idea of their personal device being recruited to help out their service provider. People may ask themselves, why would I spend my battery to relay your traffic?*" In other words, battery-limited devices must have incentives to contribute in D2D-LB by sharing their resources. A trivial plan is to adopt fixed-payment policy. Despite its apparent simplicity, this policy fails to adapt to device-dependent parameters. As such, it is incompetent to accommodate diverse device willingness-to-participate and to minimize the cost of CSP.

A conceivable design is to employ a *reverse or procurement* auction mechanism, in which multiple sellers (devices) submit bids to a single auctioneer (CSP) to provide a service (participate in D2D-LB). Then, the auctioneer evaluates all the bids and decides to purchase a subset of bids (to use their resources for D2D-LB) such that its load balancing target is achieved and *social cost* (defined in Definition 1) is minimized.

Theoretical Challenges and Approaches. Due to device battery capacity constraint, the D2D-LB auction problem turns out to be an online combinatorial auction one that is uniquely challenging and different from existing auction studies for data offloading in cellular networks (e.g., [15], [27], [39], see Table I for differences). The three key challenges and our approaches toward tackling them are as follows.

First, the D2D-LB auction problem is an online problem mainly because of the battery capacity constraint of devices. While the battery lifetime is on timescale of several hours, the inputs to the problem, including the bids' information and load-balancing requirement change on timescales that are much smaller (minutes, say). Consequently, tackling the problem emphasizes an online solution design, in which the problem is a multi-slot one whose inputs arrive online. Our general approach is first to decouple the problem into multiple single-slot problems such that their objectives are intelligently modified to capture the time-coupled structure of the problem.

Second, the D2D-LB problem even for single-slot setting is an NP-hard combinatorial problem. Our approach is to design an approximation algorithm by leveraging a primal-dual approximation framework for solving a certain type of integer problems [11]. The original primal-dual framework [11] is designed for problems with just covering constraints. However, our D2D-LB problem encounters both packing and covering constraints and it is known that problems with mixed covering and packing constraints are more challenging [30].

Third, the ultimate auction design must be dominant strategy incentive compatible [26] so that bidding according to true willingness is a dominant strategy for sellers (devices). By leveraging the celebrated Myerson result [25], we establish the truthfulness of our designed auction.

Summary of Contributions and Paper Organization. In Sec. IV, we formulate the D2D-LB auction problem and show it is NP-hard problem. In Sec. V, we propose a polynomial-time approximation algorithm for the single-slot problem. Our algorithm is a primal-dual greedy one, which chooses a set of "minimum-cost" bids to fulfill the D2D-LB target. We prove that the approximation ratio of the algorithm is 2ϕ, where $\phi \geq 1$ is a parameter capturing variations of the submitted bids. In addition, we demonstrate that our proposed algorithm guarantees truthfulness of the auction. In Sec. VI, we address the first challenge above and propose an online algorithm for the multi-slot problem based on the single-slot solution. We show the competitive ratio of our online algorithm is upper bounded by $2\phi\eta/(\eta-1)$, where $\eta \geq 1$ is a parameter determined by device limitation and bidding structure. In Sec. VII, we discuss several practical issues regarding real implementation of our proposed auction framework. In particular, we discuss about the overheads of information exchange and the approaches toward addressing interference in D2D-LB communication. In Sec. VIII, by experiments based on real-world traces, we show that our algorithm achieves near offline-optimal performance. In particular, the empirical ratio between the cost of our online solution and the offline optimal is no more than 1.44. Experimental results also show that our online solution achieves a cost saving of 66% and 45% as compared to two alternative heuristics, respectively. Finally, Sec. II reviews related literature. The system model is introduced in Sec. III, and Sec. IX concludes the paper.

TABLE I
SUMMARY AND COMPARISON OF THE PREVIOUS AND THE CURRENT AUCTION DESIGN WORKS

Reference	**Scenario**	**Underlying optimization problem**	**Competitive? (For online solution)**	**Consider battery capacity of relaying device?***	**Truthful -ness?****
Dong *et al.* [15]	WiFi offloading	Offline linear/convex	**NA**	**NA**	✓
Iosifidis *et al.* [19]	WiFi offloading	Offline convex	**NA**	**NA**	✓
Paris *et al.* [27]	WiFi offloading	Offline combinatorial	**NA**	**NA**	✓
Zhang *et al.* [34]	WiFi offloading	Online linear	✓	**NA**	✓
Zhu *et al.* [38]	D2D content sharing	Offline combinatorial	**NA**	✗	✓
Li *et al.* [23]	D2D resource sharing	Offline linear	**NA**	✗	✓
This work	D2D load balancing	Online combinatorial	✓	✓	✓

($\star$) requires to solve a multi-slot problem with multi-slot capacity constraint in online fashion, ($\star\star$) important for auction design

II. RELATED WORK

The Previous Research on D2D-LB. In the recent studies [13], [14], [24], the idea of D2D-LB has been advocated and several technical challenges have been addressed. In [24], Liu *et al.* focus on the examining the technical feasibility and practical algorithm design in three-tier LTE-Advanced networks. In [14], Deng *et al.* characterize the maximum benefits of D2D-LB in terms of peak reduction. In [13], a D2D resource allocation strategy in a three-tier heterogeneous network is presented. To the author's knowledge, there is no prior work to address the economic aspects of D2D-LB.

D2D-LB vs. Smart User Association. As mentioned in the introduction, smart user association [32] can also be applied to balance cell-level traffic. We note that D2D-LB and smart user association are complementary schemes in the sense that the CSP can simultaneously use smart user association on large timescale and D2D-LB on small timescale. Smart user association schemes normally operate on large timescale to avoid large overhead incurred by frequently associating a user from one BS to another BS [5], [32]; thus it is not designed for balancing traffic across BSs on small timescale. In contrast, D2D-LB is among adjacent devices, consumes limited power, incurs no interference, and has limited impact to the global configuration of the cellular network. These features make D2D-LB more suitable for load balancing on small timescale.

D2D-LB vs. Data Offloading; Technical Differences. Data offloading [21], mainly using WiFi infrastructure, is another popular approach to handle the exploding mobile data traffic. However, data offloading and D2D-LB are technically different schemes; while data offloading aims to exploit outband spectrum, D2D-LB targets to increase inband cellular temporal spectrum efficiency. Furthermore in D2D-LB, the CSP can ubiquitously control everything, including both D2D and user-to-BS transmissions. However, data offloading usually outsources relaying a portion of traffic to a third-party entity, that imposes unpleasant unreliability for transmissions. Therefore, D2D-LB can ensure better QoS than data offloading.

D2D-LB vs. Data Offloading; Auction Design Differences. The most related auction design studies are [15], [19], [27], [34] in WiFi mobile offloading and [23], [38] in D2D content or resource sharing scenarios. In Table I, the summary and the comparison of these works and our work are listed. Note that in our scenario, limited battery capacity of mobile devices imposes a time-coupled capacity constraint to the auction problem (see Sec. IV for details). This turns the problem into a multi-slot one that spans across the time dimension. However, the inputs to the problem are subject to timely change due to the mobility of devices and unpredictability of total traffic. Thereby, the input to the multi-slot problem arrives in online fashion, and hence, a realistic D2D-LB auction must be online. Despite elegant results, most of the previous research either in WiFi or D2D offloading scenarios [15], [19], [23], [27], [38] have focused on single-slot (offline) design spaces and ignore the temporal correlation in underlying problem. Even though in a recent study [34] the authors tackle an online problem, there is no device capacity constraint and it is not a combinatorial problem.

Moreover, D2D-LB is a combinatorial auction in which combinatorial nature is due to $0/1$-decision on purchasing a subset of the bids. This turns the problem into an NP-hard integer linear program (for proof see Theorem 1). In contrast, the auction problems in [15], [19], [23], [34] are conventional auctions (i.e., the underlying problems are either linear or convex) that are generally more tractable. Even though the problem in [27] is combinatorial, it considers a single-slot problem (thereby it fails to capture time-coupled constraints) and there is no performance guarantee for the heuristic solutions. In summary, D2D-LB enforces the auction to be an *online combinatorial* one, while none the previous work simultaneously tackle this type of auction problem.

Online Competitive Algorithms Design vs. Other Similar Solution Approaches. In addition to studies in offloading scenarios, several other papers have studied incentive mechanisms for different device-to-device scenarios in either cellular or other networks following stochastic optimization approaches [20], [22]. There is a fundamental technical difference between the solution approach in this paper and the above studies. Our approach is known as competitive analysis [10], in which neither the exact values nor the distribution of the future input is known in advance. In contrast, in [20], [22], the solutions rely on specific stochastic modeling of the input.

As in competitive analysis there is no assumptions on the stochastic modeling of future input, the online algorithm tries to compete against an adversarial input. Hence, the competitive algorithm might be conservative and not always can provide satisfactory results in practical scenarios. On the other hand, stochastic optimization approaches rely on the distribution of the input sequence. However, learning the potentially time-varying distribution in real inputs can be a formidable task.

TABLE II
SUMMARY OF NOTATIONS

Notation	Description
$\mathcal{U}$	The set of devices, ($U \triangleq \lvert\mathcal{U}\rvert$)
$\mathcal{T}$	The set of time slots, ($T \triangleq \lvert\mathcal{T}\rvert$)
$\mathcal{L}$	The set of cellular users, ($L \triangleq \lvert\mathcal{L}\rvert$)
$\mathcal{S}$	The set of adjacent BSs, ($S \triangleq \lvert\mathcal{S}\rvert$)
$a_{u,l}^t$	The amount of traffic of user l that device u can relay at t
$b_{u,l}^t$	Cost of $a_{u,l}^t$
D^t	The net D2D-LB traffic demand at time t
M_l^t	The total traffic demand of user l at slot t
C_u	The total relaying capacity of device u
C_s^t	The total relaying capacity of BS s at time t
$x_{u,l}^t$	Optimization variable, 1: device u's bid on user l and time t is successful; 0: otherwise
$y_{u,l}^t$	Optimization variable, the amount of traffic of user l that is relayed by device u at time t

III. THE D2D LOAD BALANCING AUCTION MODEL

Consider a heavy-loaded cellular base station and multiple devices in its coverage that are willing to participate in D2D-LB. A reverse auction is initiated by the CSP by soliciting the bids from the devices. Then, a subset of the bids is chosen so as to fulfill the D2D-LB target with the minimum overall cost. The key notations used in this paper are listed in Table II.

The system runs in a time-slotted manner in a time horizon of T slots. The length of each slot t and the time horizon $\mathcal{T}$ are system design parameters. A conceivable value for T might be in order of a couple of hours that is comparable to battery life of mobile devices. In this way, the devices can plan for their D2D quota (see Eq. (1)) based on the remaining battery capacity. The length of each slot is much shorter (e.g., 5-15 minutes) to capture the dynamics in devices' mobility and traffic fluctuations. We assume that CSP only knows the input to the problem for the current slot, say, 5 minutes. Beyond that, the inputs are unknown and we do not have any assumptions on the stochastic modeling of the future input. In addition, we assume that the wireless channel is time-varying and frequency-selective, but unchanged and flat during each slot. In this way, the input at each slot from the devices, the cellular users, and adjacent base stations are assumed to be constant. Note that, inputs across slots can take arbitrary values and in practice are revealed in a slot-by-slot fashion.

Devices, users, and idle base stations. Let $\mathcal{U}, (U \triangleq |\mathcal{U}|)$ be the set of devices that are available to participate in D2D-LB. Let us denote by $\mathcal{L}, (L \triangleq |\mathcal{L}|)$ as the set of cellular users. In this paper, we distinguish between *device* and *user*. By device we mean the bidder who participates in D2D-LB by forwarding the traffic that is generated by the *users*. Logically, the sets of devices and users are disjoint. Define $\mathcal{S}, (S \triangleq |\mathcal{S}|)$ as the set of idle base stations that are adjacent to the main BS (BS2 in Fig. 2, say). We assume each device $u \in \mathcal{U}$ is paired to exactly one BS $s \in \mathcal{S}$, perhaps the one with the best link quality. Denote $\mathcal{U}(s) \subseteq \mathcal{U}$ as the set of devices that are paired to BS s. The total amount of traffic that device u over time horizon $\mathcal{T}$ can load-balance is limited to its quota C_u which is set by the user based on different preferences such as remaining battery of the device [2]. Further, let M_l^t be the net traffic demand of user l at slot t, that could be either uplink or downlink traffic demand. Finally, let C_s^t be the total D2D-LB capacity of adjacent BS s. This value represents the amount of traffic that base station s is able to forward at slot t and could be obtained by subtracting the amount of traffic that is required to fulfill its own traffic from total capacity. In this way, a dedicated amount of spectrum resources is devoted to D2D-LB and there would be no interference between D2D links and device-to-BS links in the adjacent base station. Note that we assume that the spectrum resources of adjacent BSs are orthogonal and there is no interference between them. Furthermore, let D^t be the net D2D-LB demand of the main BS at time t, which is obtained by subtracting the threshold capacity of the cell from the total demand, i.e., the BS requires to load balance D^t Mb of its total demand to the adjacent under-utilized cells by leveraging the available devices.

Properties of the bids. At the beginning of slot t, device u submits multiple bids each of which consists a pair of $(a_{u,l}^t, b_{u,l}^t)$, where $a_{u,l}^t$ is the amount of traffic demand of user l (in Mb, say) that device u would like to load-balance at slot t, and $b_{u,l}^t$ is the cost of forwarding $a_{u,l}^t$. An important issue is that the D2D cost of forwarding traffic (i.e., $b_{u,l}^t$) varies for different devices based on the quality of D2D links. Indeed, the better the link quality, the lower the D2D cost. Because of different link qualities between the devices and the users, the bids are submitted separately per users. We remark that most of these different *device-dependent* cost considerations are not revealed to the CSP. Hence, this is another motivation for exploiting reverse auction in this work. Computing the bidding cost depends on device specifications (hardware, battery, etc.) and user preferences. In this paper, we assume that bidding cost is the input and the approaches on how to calculate its value is beyond the scope of this study. Finally, we assume that there is a "pre-auction phase" and a "post-auction phase", in which the inputs to the problem are preprocessed and the result of the auction realizes. We refer to Sec. VII for details.

IV. THE D2D LOAD BALANCING AUCTION PROBLEM

In a nutshell of the reverse auction, with the objective of minimizing social cost, a subset of submitted bids should be chosen such that D^t Mb of total traffic is fulfilled by the selected devices, and at the same time, the capacity constraints of adjacent BSs and devices are respected.

A. Optimization Variables

The optimization variables of the problem are as follows: (i) a binary variable $x_{u,l}^t$, associated to each bid, $x_{u,l}^t = 1$ indicates a successful bid, and $x_{u,l}^t = 0$ otherwise; and (ii) a real variable $y_{u,l}^t$ for each bid, where $y_{u,l}^t$ is the amount of traffic of user l that is load balanced by device u at slot t, indeed $0 \le y_{u,l}^t \le x_{u,l}^t a_{u,l}^t$. More specifically, after solving the auction problem, $y_{u,l}^t = 0$ for the pair of devices and users who are not selected to participate in D2D load balancing. On the other hand, $y_{u,l}^t \le a_{u,l}^t$ for the selected devices.

It is worth noting that the first binary variable turns the underlying problem into a combinatorial one that are generally more challenging to tackle as compared to conventional auctions in either linear or convex forms [15], [19], [34].

B. Constraints of the Problem

Device capacity (quota) constraint. The aggregated D2D forwarding traffic of device u over the time horizon $\mathcal{T}$ is limited by its long-term quota C_u, i.e.,

$$\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}} y_{u,l}^t \leq C_u, \quad \forall u \in \mathcal{U}. \tag{1}$$

Device quota constraint mainly depends on the battery capacity of the devices. The relationship between battery capacity and quota constraint could be calculated by energy profiling tools that estimate the battery usage of data transmission under different network conditions, link qualities, and device and battery specifications [18], [28].

Recall that the timescale of quota constraint is comparable to the battery lifetime, i.e., in order of a couple of hours. On the other hand, the D2D-LB auction decisions must be in order of minutes, because of (i) dynamics in traffic of main and adjacent base stations and (ii) dynamics in bid characteristics of devices due to their mobility. In this way, the overall problem is a time-coupled one with temporal variations in the input. From the perspective of problem formulation, this is the main differences of this work as compared to the similar auction studies, e.g., [15], [19], [27].

Device limit constraint. Due to hardware limitations of devices, we assume that each device can transmit the traffic of only one user at any given slot, even though each device can submit multiple bids each of which associated to a particular user. In this way, the number of winning bids of each device u at slot t must be less than or equal to 1, i.e.,

$$\sum_{l\in\mathcal{L}} x_{u,l}^t \leq 1, \quad \forall u \in \mathcal{U}, t\in\mathcal{T}. \tag{2}$$

User demand constraint. Total D2D forwarding traffic in user l must be less that the total traffic generated M_l^t, i.e.,

$$\sum_{u\in\mathcal{U}} y_{u,l}^t \leq M_l^t, \quad \forall t\in\mathcal{T}, l\in\mathcal{L}. \tag{3}$$

Adjacent BS capacity constraint. This constraint enforces that total D2D forwarding traffic using the capacity of BS s at slot t cannot exceed its capacity C_s^t, i.e.,

$$\sum_{u\in\mathcal{U}(s)}\sum_{l\in\mathcal{L}} y_{u,l}^t \leq C_s^t, \quad \forall s\in\mathcal{S}, t\in\mathcal{T}. \tag{4}$$

In this way, the D2D transmission is limited to the available capacity (that could be translated to the available spectrum) of each base station. In other words, if there is a vacant spectrum, an interference-free D2D link could be established, otherwise, the above constraint prevent to transmit using D2D links.

D2D traffic covering constraint. This constraint ensures that the aggregate solicited traffic by devices covers the D2D requirement D^t, i.e.,

$$\sum_{l\in\mathcal{L}}\sum_{u\in\mathcal{U}} y_{u,l}^t \geq D^t, \quad \forall t\in\mathcal{T}. \tag{5}$$

We note that D^t is calculated at the beginning of each slot with feasibility taken into account [14]. This constraint links our incentive mechanism design to the actual traffic to be balanced by D2D communication. In our solution design in Sec. V-B, it is the key to re-express this covering constraint into an equivalent expression shown in the following proposition.

Proposition 1. *Constraint* (5) *could be strengthened by the following constraint:*

$$\sum_{(u,l)\in\mathcal{B}\backslash\mathcal{A}^t} y_{u,l}^t(\mathcal{A}^t) \geq D(\mathcal{A}^t), \quad \forall\mathcal{A}^t\subseteq\mathcal{B}, t\in\mathcal{T}, \tag{6}$$

where $\mathcal{B}=\{(u,l)|u\in\mathcal{U}, l\in\mathcal{L}\}$ is the set of all bids, $\mathcal{A}^t\subseteq\mathcal{B}$ is a subset of the bids, such that total D2D traffic demand that can be satisfied by $\mathcal{A}^t$ is less than total D2D-LB demand at slot t, i.e., $\sum_{(u,l)\in\mathcal{A}^t} y_{u,l}^t < D^t$. In addition, $D(\mathcal{A}^t)$ is the residual demand of bid set $\mathcal{A}^t$ and is defined as follows: $D(\mathcal{A}^t) = D^t - \sum_{(u,l)\in\mathcal{A}^t} y_{u,l}^t$. Finally, $y_{u,l}^t(\mathcal{A}^t) = \min\{y_{u,l}^t, D(\mathcal{A}^t)\}$.

The intuition behind formulating constraint (6) is as follows. In Proposition 1, the residual demand $D(\mathcal{A}^t)$ means that even if all of the bids in the set $\mathcal{A}^t$ are chosen, the auctioneer must choose some more bids from $\mathcal{B}\backslash\mathcal{A}^t$ such that the residual D2D traffic demand $D(\mathcal{A}^t)$ is satisfied. It turns out this scenario with new residual demand as the covering constraint could be considered as another D2D auction scenario where the bids are restricted to set $\mathcal{B}\backslash\mathcal{A}^t$, instead, with $D(\mathcal{A}^t)$ as the value of covering constraint. However, in the new scenario the value of original amount for some bids might be greater than the residual demand $D(\mathcal{A}^t)$. This is addressed by introducing $y_{u,l}^t(\mathcal{A}^t)$ in Proposition 1. In Lemma 1, we express how this reformulation can be incorporated to reduce the integrality gap of the corresponding underlying problem, leading to an elegant structure to design primal-dual approximation algorithm.

Causality constraints. Finally, by defining $a_{u,l}^t(\mathcal{A}^t) = \min\{a_{u,l}^t, D(\mathcal{A}^t)\}$, we have the following causality constraints

$$0 \leq y_{u,l}^t \leq a_{u,l}^t x_{u,l}^t, \quad \forall u\in\mathcal{U}, l\in\mathcal{L}, t\in\mathcal{T}, \tag{7}$$
$$0 \leq y_{u,l}^t(\mathcal{A}^t) \leq a_{u,l}^t(\mathcal{A}^t) x_{u,l}^t, \quad \forall u,l,t,\mathcal{A}^t\subseteq\mathcal{B}, \tag{8}$$

where constraint (7) ensures that the if device u is selected to forward the traffic of user l at slot t, i.e., $x_{u,l}^t = 1$, its forwarding amount $y_{u,l}^t$ is below the bidding amount $a_{u,l}^t$. Otherwise, i.e., $x_{u,l}^t = 0$, it forces $y_{u,l}^t$ to be 0. Similarly, the constraint (8) represents the same concept using the set notation introduced in Proposition 1.

C. The D2D-LB Auction Problem

With the goal of minimizing *social cost* (defined in Definition 1), we formulate the D2D-LB auction problem ($\mathsf{D2DAuc}$).

Definition 1. *(Social welfare and social cost) Under truthful bidding (see Sec. V-D), the social welfare in a D2D-LB auction is the aggregate utility of the CSP, i.e., $-\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}\sum_{u\in\mathcal{U}} z_{u,l}^t x_{u,l}^t$, where $z_{u,l}^t$ is the payment to device u in user l at slot t, and the aggregate utility of bidding devices, i.e., $\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}\sum_{u\in\mathcal{U}}\left(z_{u,l}^t - b_{u,l}^t\right) x_{u,l}^t$. Payments* *between the CSP and devices cancel themselves, and the social welfare is equal to* $-\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}\sum_{u\in\mathcal{U}} b^t_{u,l}x^t_{u,l}$*. Maximizing the social welfare is equivalent to minimizing the social cost, i.e., total system cost, which in turn is equivalent to minimize* $\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}\sum_{u\in\mathcal{U}} b^t_{u,l}x^t_{u,l}$ *under truthful bidding.*

Under the assumption of truthful bidding, problem D2DAuc is formulated as follows

$$\begin{aligned} \textsf{D2DAuc}: \quad & \min && \sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}\sum_{u\in\mathcal{U}} b^t_{u,l}x^t_{u,l} && \text{(9a)}\\ & \text{s.t.} && \text{Constraints (1)-(4), (6)-(8)} && \text{(9b)}\\ & \text{vars.} && x^t_{u,l}\in\{0,1\}, y^t_{u,l}\in\mathbf{R}^+. && \text{(9c)} \end{aligned}$$

D2DAuc is a mixed integer linear program that belongs to capacitated covering problems[1] [12].

Theorem 1. *Problem* D2DAuc *is NP-hard.*

As a consequence of Theorem 1, direct application of the VCG mechanism [26] as a well-established auction that ensures truthful bidding is computationally infeasible, since it requires the optimal solution to the underlying problem. An alternative way is to relax the integer constraints and solve the corresponding relaxed LP. The following Lemma characterizes the integrality gap of problem D2DAuc with original formulation of covering constraint as in Eq. (5).

Lemma 1. *The integrality gap of* D2DAuc *with original covering constraint* (5) *is* $\geq \max_{t\in\mathcal{T}} D^t$.

Lemma 1 analytically motivates the re-expression of constraint (5) as Eq. (6). Observe that for problem D2DAuc with Eq. (6) instead of Eq. (5), the given LP solution is no longer feasible for the instance with bad integrality gap in the proof of Lemma 1 (see [31, pp.179] for in-depth discussion).

Finally, note that constraints (2)-(8) are separable in time. However, device quota constraint (1) is coupled over the time horizon, thereby D2DAuc is coupled over time. On the other hand, the problem data, such as the D2D-LB requirements and bids' information arrive online. Putting together, tackling D2DAuc requires online solution design. What exacerbates the problem is that even singles-slot problem (by neglecting Constraint (1)) is still NP-hard (see the proof of Theorem 1). In the next, we tackle the single-slot D2DAuc (named as "sD2DAuc") and devise a polynomial approximation algorithm. Then, in Sec. VI, we leverage the single-slot algorithm and devise an online algorithm for the general problem.

V. Truthful Approximation Algorithm Design for Single-Slot Problem

Our auction algorithm design in this section (for single-slot scenario) and in the next section (for online multi-slot scenario) aims to achieve the following goals: (i) to be computationally efficient—the solution can scale with the problem size; (ii) to promote truthful bidding—to prevent devices to game the system and to simplify both the bidding strategy and the auction design; (iii) to design an approximation algorithm for single-slot scenario—to guarantee sound performance against the optimum; and (iv) to design competitive online algorithm for online scenario—to achieve small loss as compared to the single-slot scenario.

[1]The problem of $\min\{bx \mid Ax\geq d, 0\leq x\leq c, x\in\mathbf{Z}^+\}$, where b, A, d, c are nonnegative, is a capacitated covering problem.

A. Formulating Single-Slot Problem

We first formulate single-slot problem sD2DAuc as:

$$\begin{aligned} & \textsf{sD2DAuc} && \\ \min \quad & \sum_{l\in\mathcal{L}}\sum_{u\in\mathcal{U}} c_{u,l}x_{u,l} && \text{(10a)}\\ \text{s.t.} \quad & \sum_{l\in\mathcal{L}} x_{u,l}\leq 1, \quad \forall u\in\mathcal{U}, && \text{(10b)}\\ & \sum_{u\in\mathcal{U}} y_{u,l}\leq M_l, \quad \forall l\in\mathcal{L}, && \text{(10c)}\\ & \sum_{u\in\mathcal{U}(s)}\sum_{l\in\mathcal{L}} y_{u,l}\leq C_s, \quad \forall s\in\mathcal{S}, && \text{(10d)}\\ & \sum_{(u,l)\in\mathcal{B}\setminus\mathcal{A}} y_{u,l}(\mathcal{A})\geq D(\mathcal{A}), \forall\mathcal{A}\subseteq\mathcal{B}, && \text{(10e)}\\ & 0\leq y_{u,l}\leq a_{u,l}x_{u,l}, \quad \forall u,l, && \text{(10f)}\\ & 0\leq y_{u,l}(\mathcal{A})\leq a_{u,l}(\mathcal{A})x_{u,l}, \quad \forall u,l,\mathcal{A}, && \text{(10g)}\\ \text{vars.} \quad & x_{u,l}\in\{0,1\}, y_{u,l}\in\mathbf{R}^+, \forall u\in\mathcal{U}, l\in\mathcal{L}. && \end{aligned}$$

As compared to problem D2DAuc, the device packing constraint is neglected in problem sD2DAuc and the superscript t is dropped from the notations, since the problem is solved for a specific $t\in\mathcal{T}$. In addition, the cost $b_{u,l}$ is replaced by the scaled cost $c_{u,l}$ according to the remaining D2D capacity of device u (the rationale is mentioned in detail in Section VI).

In the next subsection, we propose a primal-dual greedy approximation algorithm for solving problem sD2DAuc. The algorithm iteratively updates both primal and dual variables and the approximation analysis is based on duality property. Thereby, in the following we formulate dual problem associated to LP relaxed version of sD2DAuc (i.e., $x_{u,l}\in[0,1]$)

$$\begin{aligned} \max \quad & \sum_{\mathcal{A}\subseteq\mathcal{B}}\gamma(\mathcal{A})D(\mathcal{A}) - \sum_{u\in\mathcal{U}}\lambda_u - \sum_{l\in\mathcal{L}}\zeta^l M^l - \sum_{s\in\mathcal{S}}\mu_s C_s \\ \text{s.t.} \quad & \sum_{\mathcal{A}\subseteq\mathcal{B}:(u,l)\notin\mathcal{A}} \rho_{u,l}(\mathcal{A})a_{u,l}(\mathcal{A}) + a_{u,l}\nu_{u,l} - \lambda_u\leq c_{u,l}, \\ & \forall u,l && \text{(11a)}\\ & \gamma(\mathcal{A})-\rho_{u,l}(\mathcal{A})\leq 0, \forall u,l,\mathcal{A}, && \text{(11b)}\\ & \zeta^l + \sum_{s:u\in\mathcal{U}(s)}\mu_s + \nu_{u,l}\geq 0, \quad \forall u,l, && \text{(11c)}\\ \text{vars.} \quad & \lambda_u\geq 0, \zeta^l\geq 0, \mu_s\geq 0, \\ & \gamma(\mathcal{A})\geq 0, \rho_{u,l}(\mathcal{A})\geq 0, \nu_{u,l}\geq 0, \end{aligned}$$

where dual variables λ, ζ, μ, γ, ν, and ρ correspond to primal constraints (10b), (10c), (10d), (10e), (10f), and (10g), respectively.

In Sec. V-B, we devise an approximation algorithm which is inspired by a primal-dual framework that has been proposed in [11] for capacitated covering problems. Then, in Sec. V-C, a theoretical bound is achieved for the approximation ratio of the proposed algorithm, and we investigate the truthfulness of the proposed solution in Sec. V-D. The vigilant readers may notice that some similar designs are recently proposed for auction design in demand response in co-location data centers [33] and storage-assisted smart grids [36], [37], and resource pooling in cloud storage systems [35]. However, in this work we need to tackle further challenges initiated from packing constraints (i.e., Eqs. (10d)-(10c)) unique to our problem.

B. Approximation Algorithm Design

The algorithm is demonstrated as Algorithm 1. In a nutshell of our algorithm, through an iterative procedure and in a greedy manner, the "minimum-cost" bids are chosen until the selected bids can cover the D2D-LB demand D. More specifically, in **Initialization** block, in Lines 2-6, both primal and dual variables are initialized (note that we just mentioned dual variables whose values are changed in the algorithm, others are supposed to be initialized to zero and investigated in Lemma 4). We specifically remark that the set $\mathcal{I}$ is the set of the selected pairs $(u,l), \forall u, l$, and by terminating the while loop, the bids in the set $\mathcal{I}$ are chosen as the winning bids.

Algorithm 1: Single-Slot Algorithm, for $t \in \mathcal{T}$

1 **Initialization**
2 $\mathcal{U}^{\mathsf{sel}} = \emptyset$ `// the set of selected devices`
3 $\mathcal{I} = \emptyset$ `// the set of selected bids`
4 $x_{u,l} = 0, y_{u,l} = 0, \forall u, l$`// primal variable`
5 $\gamma(\mathcal{A}) = 0, \forall \mathcal{A} \subseteq \mathcal{B}$ `// constraint` (10e)
6 $\hat{c}_{u,l} = c_{u,l}, \forall u, l$`// scaled cost variable`
7 **while** $\mathcal{U}^{\mathsf{sel}} \neq \mathcal{U}$ ***and*** $D(\mathcal{I}) > 0$ **do**
8 $\quad \hat{c}_{u,l} = \hat{c}_{u,l} - \gamma(\mathcal{I}) a_{u,l}(\mathcal{I}), \quad \forall u, l \notin \mathcal{I}$
9 $\quad M_l(\mathcal{I}) = \left[M_l - \sum_{(u,l)\in\mathcal{I}} y_{u,l}\right]^+, \quad \forall l \in \mathcal{L}$
10 $\quad C_s(\mathcal{I}) = \left[C_s - \sum_{(u,l)\in\mathcal{I}: u\in\mathcal{U}(s)} y_{u,l}\right]^+, \quad \forall s \in \mathcal{S}$
11 $\quad \hat{a}_{u,l}(\mathcal{I}) = \min\{a_{u,l}(\mathcal{I}), M_l(\mathcal{I}), C_s(\mathcal{I})\}$
12 $\quad (u^\star, l_u^\star) = \arg\min_{u\in\mathcal{U}\setminus\mathcal{U}^{\mathsf{sel}}, l} \left\{\frac{\hat{c}_{u,l}}{\hat{a}_{u,l}(\mathcal{I})}\right\}$
13 $\quad \gamma(\mathcal{I}) = \frac{\hat{c}_{u^\star,l_u^\star}}{a_{u^\star,l_u^\star}(\mathcal{I})}$
14 $\quad x_{u^\star,l_u^\star} = 1$
15 $\quad y_{u^\star,l_u^\star} = \hat{a}_{u,l}(\mathcal{I})$
16 $\quad \mathcal{I} = \mathcal{I} \cup (u^\star, l_u^\star)$
17 $\quad \mathcal{U}^{\mathsf{sel}} = \mathcal{U}^{\mathsf{sel}} \cup u^\star$
18 **end**

The second block is the **while** loop. The stop conditions are two-fold. The first one prevents infinite loop. The second condition is the one that terminates the loop when the selected bids can cover the demand, i.e., constraint (10e) is satisfied. In Line 8, we adjust the cost of the remaining bids based on their effective traffic amount. Modification in Line 11 based on the adjustments in Lines 9 and 10 is required to respect constraints (10c) and (10d). As the result of this modification, the unit costs of bids that require saturated users/adjacent BSs approaches infinity, so they do not selected in the greedy algorithm. Note that this modifications are not appeared in the original framework [11] and it is unique to this work. Then, in Line 12 the bid with the minimum ratio between the scaled cost and the effective traffic amount (i.e., $\hat{a}_{u,l}(\mathcal{I})$) is chosen and this bid is added to the set of selected bids in Lines 16-17. Moreover, the corresponding primal variables are set in Lines 14-15. In Line 13, the dual variable is set such that the dual constraint becomes tight for the selected bid. Finally, it is straightforward to show that the time complexity of Algorithm 1 is $O(UL)$, which is polynomial.

C. Approximation Ratio Analysis

In this subsection, we analyze approximation ratio of Algorithm 1. Our approach is to use the duality property to derive a bound for approximation algorithm. In particular, let p and d be the primal and dual values obtained by Algorithm 1, respectively. Furthermore, let $p^\star$ as the optimal value of problem $\mathsf{sD2DAuc}$. Indeed, $p \geq p^\star$, and by duality property, $d \leq p^\star$. The target is to find an α so as to $\alpha d \geq p$, thereby, $p/p^\star \leq p/d \leq \alpha$, hence, Algorithm 1 is α-approximate. Our analysis consists of three steps. First, Lemma 2 shows that Algorithm 1 generates a feasible solution to problem $\mathsf{sD2DAuc}$. Second, Lemma 4 proves dual feasibility, and finally, Theorem 2 provides approximation factor.

Lemma 2. *Algorithm 1 terminates with a feasible solution for primal problem $\mathsf{sD2DAuc}$.*

Before proceed to prove dual feasibility, we introduce the following Lemma. Intuitively, the following Lemma mention that constraint (11a) becomes tight for any selected bid $(u,l) \in \mathcal{I}$ by the end of Algorithm 1.

Lemma 3. $\sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \gamma(\mathcal{A}) a_{u,l}(\mathcal{A}) = c_{u,l}, \forall (u,l) \in \mathcal{I}$.

The next step is to prove dual feasibility of the solution computed by Algorithm 1. Even if the primal solution is feasible during the execution, the dual is not necessarily so. But, by an adjustment in dual variables the dual solution becomes feasible. The purpose of dual adjustments in Lemma 4 is to make the dual solution feasible through scaling by a carefully chosen factor. Such posterior dual scaling is known as *dual fitting* in the primal-dual approximation literature, and has proven effective in finding good approximation ratios in algorithm design [30].

Lemma 4. *The dual solution obtained by Algorithm 1 is a feasible solution to dual problem of $\mathsf{sD2DAuc}$ by setting/adjusting the dual variables as follows*

- $\lambda_u = 0; \forall u \in \mathcal{U}$, *constraint* (10b),
- $\zeta^l = 0; \forall l \in \mathcal{L}$, *constraint* (10c),
- $\mu_s = 0; \forall s \in \mathcal{B}$, *constraint* (10d),
- $\nu_{u,l} = 0; \forall u, l$, *constraint* (10f),
- $\gamma(\mathcal{I}) = \gamma^{\mathcal{I}}/\phi$, *constraint* (10e)*, where*

$$\phi = \max_{u,u'\in\mathcal{U},l,l'\in\mathcal{L}} \left\{\frac{c_{u,l}}{c_{u',l'}}, \frac{c_{u,l}a_{u',l'}}{c_{u',l'}a_{u,l}}\right\}, \tag{12}$$

- $\rho_{u,l}(\mathcal{A}) = \gamma(\mathcal{A}); \forall u, l, \mathcal{A} \subseteq \mathcal{S}$, *constraint* (10g).

Theorem 2. *Algorithm 1 is an α-approximation algorithm, where $\alpha = 2\phi$, where $\phi \geq 1$ is defined in Eq.* (12).

Approximation ratio of Algorithm 1 is dependent on the value of ϕ in Eq. (12) which is defined as the maximum ratio between either the exact or the normalized cost values of any two submitted bids. Note that in a scenario that user devices' specification and connection qualities are more homogeneous and devices are almost equally willing to participate in D2D-LB, then ϕ is expected to approaches 1 and the approximation ratio in Theorem 2 approaches 2.

D. Truthfulness Analysis

In Sec. IV, we formulated the problem of D2D-LB auction with truthful bidding assumption. In auction mechanism design, it is critical to promote truthful bidding, to ensure that bidding true costs is a dominant strategy for devices. In this way, devices are prevented from gaming the system, thereby both bidding strategy and auction design are simplified. In the previous algorithm design, we ignored *truthful* issues on how to make sure that devices announce their true values of cost, which is the goal of this section.

Definition 2. *An auction is truthful if for every device u, truth-telling of cost values is a dominant strategy, i.e., declaring the true costs ($b_{u,l}^t$) always maximizes a device's utility, regardless of how the other devices submit their bids.*

Definition 3. *An auction is individual rational if each device obtains a non-negative utility by participating in the auction.*

For the details, we refer to [26]. The following celebrated result by Myerson [25] is the key in truthful analysis of reverse auction mechanisms.

Lemma 5. *[7], [25] A reverse auction is truthful if and only if: (i) $x_{u,l}, \forall u, l$ is monotonically non-increasing in costs $c_{u,l}$, i.e., $\forall u \in \mathcal{U}, \forall l \in \mathcal{L}$, if $c_{u,l} \leq c'_{u,l}$, $a_{u,l} = a'_{u,l}$, and $x'_{u,l} = 1$, then $x_{u,l} = 1$. (ii) winners are paid threshold payments, i.e., $P_u = b_{u,l} x_{u,l} + \int_{b_{u,l}}^{\infty} x_{u,l} db$.*

Theorem 3. *If at iteration τ, the auctioneer pays to the selected device $u^\star$ according to*

$$P_{u^\star} = c_{u^\star, l_u^\star} + a_{u^\star, l_u^\star}(\mathcal{I}(\tau))\left(\gamma(\mathcal{I}(\tau+1)) - \gamma(\mathcal{I}(\tau))\right), \quad (13)$$

the auction results computed by Algorithm 1 is truthful.

Lemma 6. *The payment based on Eq.* (13) *makes Algorithm 1 individually rational.*

VI. The Online Algorithm Design

In this section, we design an online algorithm for solving multi-slot (online) D2DAuc. We briefly discuss the challenge and our idea as follows. In the online setting at time slot t, only the bids that are submitted at and before t are revealed and the winning bids must be determined without knowing the future bids. As mentioned in Sec. IV, D2DAuc is coupled over time due to device quota constraint (1). If the submitted bid of a device is successful in a particular slot, its residual D2D quota decreases, thereby, its chance to participate in the incoming rounds decreases because of insufficient remaining capacity. Consequently, the BS may have to cover D2D demand from expensive alternatives, whereas by scattering the winning bids intelligently on temporal domain, the cost could be reduced. Our idea is to scale up the cost of the devices based on their residual capacities—the smaller the residual capacity, the larger the scale up factor.

Algorithm 2: The Online Algorithm

1 **Initialization**
2 $\kappa_u^t = 0, \quad \forall u \in \mathcal{U}, t \in \mathcal{T}$
3 $\eta = \max_{u \in \mathcal{U}, l \in \mathcal{L}, t \in \mathcal{T}} \left\{ \frac{C_u}{a_{u,l}^t} \right\}$
4 **foreach** $t \in \mathcal{T}$ **do**
5 $\quad c_{u,l}^t = b_{u,l}^t + a_{u,l}^t \kappa_u^{t-1}, \quad \forall u \in \mathcal{U}, l \in \mathcal{L}$
6 $\quad \mathcal{I}$ = the selected set obtained by executing Algorithm 1 and adjusted costs $c_{u,l}^t, \forall u \in \mathcal{U}$ as the input.
7 $\quad \kappa_u^t = \kappa_u^{t-1}, \forall u \in \mathcal{U}$
8 $\quad \kappa_u^t = \kappa_u^{t-1}\left(1 + \frac{a_{u,l_u}^t}{\alpha C_u}\right) + \frac{b_{u,l_u}^t}{\alpha \eta C_u}, \forall u \in \mathcal{I}$
9 **end**

The detail of the algorithm is listed as Algorithm 2. Our main goal in Algorithm 2 is to scale-up the original cost of winner devices according to their remaining capacity. Toward this, we first define variable κ_u^t to be used to adjust the cost of device u as the input to the single-slot Algorithm 1 at its t-th execution. Indeed, $\kappa_u^0 = 0$, since in the beginning the remaining capacity of all the devices are $C_u, \forall u \in \mathcal{U}$. As time goes ahead, the remaining capacity of winner devices decreases, thereby as depicted in Line 8, κ_u^t increases (the rationale behind this specific increase is explained later). Then the cost of the winner devices ($(u, \star) \in \mathcal{I}$) is scaled up in the next round according to Line 5. Remark that the cost of the other devices remains intact (Line 7).

We now explain the increase in Line 8 in detail. The variable κ_u^t is scaled up (as compared to its previous value κ_u^{t-1}) for any selected device u, such that the amount of increase depends on its previous value, κ_u^{t-1} (that captures the aggregate scale up in the previous slots) and its current slot characteristics, i.e., a_{u,l_u}^t and b_{u,l_u}^t. In addition, two fixed parameters α (which is the approximation factor of Algorithm 1) and η (defined in Line 3) are incorporated to scale κ_u^t. By the cost adjustment in Line 5, variable κ_u^t could be interpreted as the unit price of using residual D2D-LB capacity of device u. As such, the increase in κ_u^t at each time slot can impact on the final result of the single-slot Algorithm 1 because adjusted cost is the input to that algorithm. In this way, if we set κ_u^t to a large value, it makes the Algorithm 1 too conservative in selecting the devices with low residual capacity, which may result in wasting resource of cost-effective devices at the end of time horizon. On the other hand, if we set κ_u^t to a small values, it comes at the risk of consuming the capacity of cheap devices too early, which may force the provider to purchase the bids of expensive devices later on. Consequently, it is important to scale up κ_u^t such that the importance of residual capacity is taken into account properly. We set the update equation for κ_u^t as in Line 8, which is the key in competitive analysis of Algorithm 2 and also works well in experiments discussed in Sec. VIII.

The results in Lemmas 7-8 prove the primal and dual feasibility of Algorithm 2 which are required as the competitive analysis of the algorithm (Theorem 4).

Lemma 7. *Algorithm 2 generates a feasible solution for problem D2DAuc.*

Lemma 8. *By setting $\kappa_u = \kappa_u^T$, Algorithm 2 generates a feasible solution for dual problem of problem D2DAuc, where κ_u is the dual variable associated to constraint* (1).

Theorem 4. *The competitive ratio of Algorithm 2 is $\frac{\alpha\eta}{\eta-1}$, where α is the approximation factor of Algorithm 1 and $\eta \geq 1$ is defined in Line 3 of Algorithm 2.*

The competitive ratio depends on the value of η that is defined as the maximum ratio between the device quota and the submitted bid amounts of any device. In this way, when $\eta \to \infty$, that is, the submitted amount in each time slot is much smaller than total device quota (i.e., $a_{u,l}^t \ll C_u, \forall u, l, t$), then, the competitive ratio approaches α, which means that (i) the online loss is zero as compared to the single-slot algorithm, and (ii) when $\phi \to 1$ (see Eq. (12) and the remark after Theorem 2), the competitive ratio is 2.

VII. Discussion

A. Pre-processing and Post-processing Phases of the Auction

There is a pre-auction phase in our model in which the information required for the auction must be exchanged between the bidders (participant devices) and the auctioneer (CSP). First, the input parameters of our model such as M_l^t, C_s^t, and D^t are calculated at the beginning of each slot, by taking into account the feasibility of supporting demand D^t by D2D-LB subject to device limitations and the required peak-traffic reduction. This can be done by adapting the D2D-LB scheme in [14]; we skip the details and focus on incentive mechanism design. Then, the BS announces the information of neighborhood cellular users to the bidders. This information is required for the devices to announce their bids. Finally, the bidders announce their bid information including the bidding cost and amount per each cellular user, separately. These steps are considered as the pre-auction phase in our model.

In the post-auction phase, at each slot t, the selected devices by the auction are scheduled by the CSP to relay D2D-LB requirement to the adjacent cells. Several engineering issues, such as authentication methods, seamless load-balancing of traffic of existing sessions, and setting up D2D links, can be addressed by adapting the techniques described in [8], [15]. In summary, similar to any other approach that relies on D2D communication, an information exchange is required between the devices and the base station. We refer to [8], [16] and references therein for the possible approaches for the information exchange and the signaling overhead.

B. Interference in D2D-LB

Generally speaking, D2D communication can be established using either cellular licensed spectrum, termed as *inband* D2D, or unlicensed spectrum, termed as *outband* D2D (e.g., through WiFi direct or Bluetooth) [8]. The inband D2D is further divided into *underlay* inband D2D and *overlay* inband D2D [8]. In underlay inband the spectrum is shared with the BS that may lead to interference. In overlay inband, a portion of licensed cellular spectrum is dedicated to D2D, thereby there is no interference between D2D links and the BS.

Our advocated idea in D2D-LB falls into the category of overlay inband D2D. More specifically, D2D-LB dynamically uses *the vacant spectrum resources* of adjacent idle BS to forward the traffic of a heavy-loaded BS. Hence, D2D transmission in D2D-LB is in the category of a dynamic dedicated overlay inband D2D, since it uses the licensed spectrum of adjacent cells. The interference problem in D2D-LB, however, must be investigated in both heavy-loaded BS and also in the adjacent BS whose vacant spectrum is used to forward the traffic. First, interference problem in heavy-loaded BS is automatically resolved because D2D-LB uses the vacant spectrum of adjacent BS which has no overlap with the spectrum of the main heavy-loaded BS. More specifically, the spectrum of adjacent base stations do not overlap, hence, there is no interference between adjacent cells. Second, the interference between D2D communication and the adjacent idle base station is also captured in our model by introducing D2D-LB capacity constraint in Eq. (4).

VIII. Simulation Results

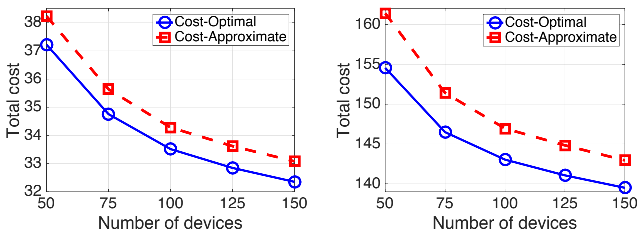
A. Overview of Traces and Parameter Settings

We use traffic traces from Smartone [3], a major cellular provider in a Hong Kong. More specifically, we use the traces of total traffic of 374 cells covering an area of 22 KM^2 of a metropolitan district to estimate total D2D traffic demand (D^t) and adjacent cell D2D-LB capacity (C_s^t). The length of each slot is set to 15 minutes and time horizon T is set to 8 hours for online scenario. Based on the measurement results in [14], we assume that at each slot 25% of total traffic demand have to be load-balanced through D2D-LB, on average, i.e, $D^t = 2.5 \pm 0.5$ Gb. We assume 4 adjacent under-utilized cells in neighborhood ($S = 4$). The number of bidding devices is $[50, 150]$ per base station and varies in different runs and the devices are randomly assigned to exactly 1 adjacent BS. This number of users are reasonable for around 5 cells (including the main cell and the other adjacent cells) in highly crowded district. The amounts of bids are generated uniformly over $[50, 150]$ Mb, and the cost values are generated uniformly over $[\$0.5, \$1.5]$, which is comparable to the common service plan tariffs for the CSP (e.g., $\$10/$Gb). Finally, the number of users are in order of devices and each devices announces on average 5 bids based on the possible pairs of device-user.

Gurobi MILP solver [1] is used to calculate the offline optimal solution and hence the performance of our proposed algorithms is evaluated as compared to the optimal. In addition, in Sec. VIII-D, we compare the performance of our algorithm to two heuristics. Finally, each data point of the figures belongs to the average values (statistical values for boxplots) of 100 runs with 95% confidence interval, where each run is a different randomly generated scenario.

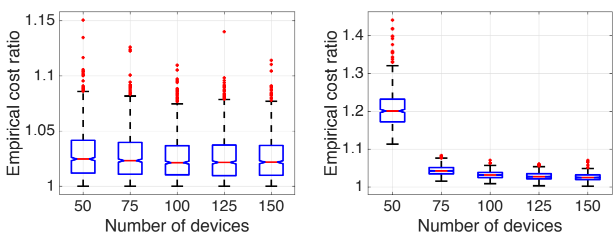
B. The Effect of the Number of Devices

Purpose. This set of experiments is devoted to compare the proposed algorithms to the optimum and to investigate the effect of the number of D2D devices (submitted bids) in final



(a) Single-slot scenario (b) Online scenario

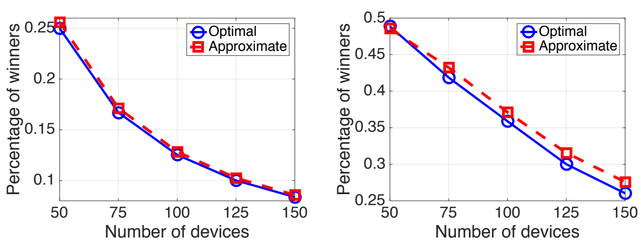
Fig. 3. The effect of number of devices on total cost



(a) Single-slot scenario (b) Online scenario

Fig. 4. The effect of number of devices on empirical cost ratio

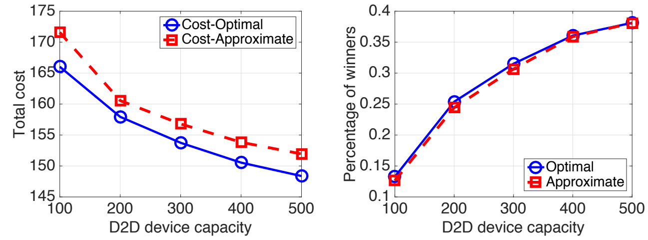
cost. Figs. 3, 4, and 5 depict total cost, empirical cost ratio (cost of approximation algorithms over the offline optimum), and the percentage of winners, for different numbers of devices, for both single-slot and multi-slot scenarios.



(a) Single-slot scenario (b) Online scenario

Fig. 5. The effect of number of devices on the % of winners

Observations. We report four notable observations. *First,* results in Fig. 3(a) for single-slot scenario, and Fig. 3(b) for online scenario depict that total cost obtained by Algorithms 1 and 2 are slightly higher than the optimal cost (on average $\approx \times 1.03$ and $\approx \times 1.07$ of the optimum for the single-slot and online scenario, respectively). These results signify that the proposed approximation algorithms can approach the optimal cost much better than those bounds obtained by theoretical results (i.e., competitive ratio of 2 in the case that $\phi \to 1$ and $\eta \to \infty$, see Theorem 4). *Second,* results in Fig. 3 show that when the number of D2D devices increases, total cost decreases. This is reasonable since the auctioneer have higher freedom to choose the more cost-effective bids as the number of devices grows. Indeed, our approximation results depict the same behavior. *Third,* the results in Fig. 4 show that cost ratios are close to 1 and also as the number of devices increases the empirical ratio for both cases decreases. Note that the worst ratio obtained is 1.44 that is happened in online case when the number of devices is 50. However, as number of devices grows to 100 this ratio is at most 1.08. *Fourth,* we demonstrate the percentage of winner devices in Figs. 5. The behavior is rationale since the probability of winning is reduced when the number of bidders increases.



(a) Total cost (b) Percentage of winners

Fig. 6. The effect of capacity of devices

C. The Effect of the Capacity of Devices

Purpose. In this experiment we investigate the effect of the capacity of devices in total cost and percentage of winners obtained by our online algorithm. Recall that our modeling captures the limitation of the battery of devices by introducing device quota constraint (1). In this experiment, we change the capacity of devices from 100 to 500 Mb and execute our algorithm and the optimal offline solution.

Observations. The result in Fig. 6(a) shows that, as the capacity of devices increases, total cost decreases. The reason is that with the increase in the capacity of devices, each winning device can contribute more in load balancing, thereby, the total cost is decreased. On the other hand, Fig. 6(b) shows that for both offline optimal solution and our online algorithm, as device capacity increases, the percentage of winners also increases. This is also reasonable since with the increase in the capacity, more devices can participate in auction design during the time horizon, thereby more devices can potentially be chosen during different slots.

D. Comparison with Alternative Solutions

Purpose. In this experiment, we focus on the single-slot setting and compare the performance of Algorithm 1 with two baseline algorithms: *simple greedy* algorithm and *random* algorithm. In the simple greedy heuristic, the devices are picked based on their original cost values in ascending order until the D2D requirement is fulfilled. In the random algorithm, we randomly select devices one-by-one to participate in D2D-LB until we have enough devices to serve the load balancing demand. We intentionally choose random algorithm to demonstrate that without proper cost minimizing algorithm, the cost of the CSP could be very large.

Observations. We summarize the comparison in Table III, where the cost column is total cost of all the three algorithms and the ratio column is the ratio of total cost of different algorithms to the optimum. Again, we observe that our approximation algorithm (Algorithm 1) is near-optimal (with average cost ratio of 1.1), outperforming both the simple greedy algorithm (with average cost ratio of 1.72) and the random algorithm (with average cost ratio of 3.4). The reason is as follows. The simple greedy algorithm only takes the bid

TABLE III
RESULTS OF THE COMPARISON SCENARIO

	Number of Devices					
	100		150		200	
	Cost	**Ratio**	**Cost**	**Ratio**	**Cost**	**Ratio**
Algorithm 1	32.77	1.13	29.38	1.12	28.55	1.07
Simple greedy	51.73	1.70	44.90	1.76	41.78	1.71
Random	95.24	3.10	96.69	3.81	97.5	3.30

cost into account, but is oblivious to the other information, including the biding amount, D2D traffic demand distribution, and adjacent BSs' capacity. Even worse, the random algorithm is oblivious to all information.

IX. CONCLUSIONS

This paper studied an important, yet open, challenge on how to incentivize devices to participate in D2D-LB paradigm. The underlying problem is formulated following a multi-slot online procurement auction framework. The objective is to minimize social cost while satisfying the D2D-LB requirement. We showed the formulated problem is NP-hard and standard LP relaxation approach may give arbitrarily bad performance. We then proposed an approximation algorithm and an online algorithm that work together to solve the problem in polynomial time with decent performance guarantee. Our algorithm design also ensures truthfulness of the auction, which is a highly desired feature in auction design. Observations on extensive trace-driven simulations demonstrated that our proposed approximation algorithms achieve near offline-optimal performance. As future work, we plan to extend our results to joint problem of scheduling and bid winner determination.

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Appendix

A. The Details of the Statistical Study of the Motivation of D2D Load Balancing

In this Appendix, we explain the details of the measurement whose result is depicted in Fig. 1. We first overview our data traces. The dataset contains the traffic data of 30 days of 194 BSs covering an area of 22km^2. The coverage area (cell radius) of each base stations is 500m. Thus, BS i and BS j are adjacent if their distance is no greater than 500m.

The goal is to evaluate the correlation of the traffic pattern of the adjacent base stations, by computing their (Pearson) correlation coefficients [9]. For each BS $i \in [1, 194]$ and each day $d \in [1, 30]$, we calculate the correlation coefficient of the traffic of BS i in day d, denoted as $R(i, d)$, which means that the traffic of any adjacent base station of BS i and that of BS i has a correlation coefficient at least $R(i, d)$. In Fig. 1(a), the empirical cumulative distribution function (CDF) of all per-BS per-day correlation coefficients, i.e., $\{R(i, d) : i \in [1, 194], d \in [1, 30]\}$, is shown. The result demonstrates that more than 50% of adjacent BSs has correlation coefficients that are less than 0.2, and also the overall average correlation coefficient is only 0.18. That is, the traffic between adjacent BSs has a relatively low correlation and thus we have ample room to reduce the peak traffic by load balancing among the base stations. To see more concretely what kind of traffic pattern will have a correlation coefficient 0.18, we show the traffic of a sampled pair of adjacent BSs in Fig. 1(b). It can be seen that the peak traffic of BS1 and BS2 occur at different epochs, suggesting that indeed the traffic from the congested cell could be shifted to the less-congested cell to reduce the peak traffic.

B. Proof of Theorem 1

Proof. The proof is done by considering $T = 1, L = 1, S = 1$, which turns the problem into the minimum knapsack problem, which is an original NP-hard problem [31]. □

C. Proof of Lemma 1

Proof. Consider $T = 1, L = 1, S = 1, D^1 = D$, and two devices where $C_1 = D-1, C_2 = D, b_1 = 0, b_2 = 1$. The only feasible integral solution is to choose device 2 with total cost 1. However, solution $x_1 = 1, x_2 = 1/D$ is feasible for LP relaxation and total cost is $1/D$. Hence, the integrality gap is at least $1/(1/D) = D$, where D could be $\max_{t\in\mathcal{T}} d_t$. □

D. Proof of Lemma 2

Proof. To prove, we show that (i) the integer constraint is respected. This is straightforward, since the values of $x_{u,l}$ are initialized to be 0, and in Line 14, the corresponding bid of the selected device is set to be 1, thereby, the integer constraint is never violated. (ii) Constraint (10b) is respected. This is true, since by selecting a device, this device is added to the set $\mathcal{U}^{\mathsf{sel}}$ in Line 17, and in the next loop, the next bid is selected among the set of the other devices, i.e., $u \in \mathcal{U}\backslash\mathcal{U}^{\mathsf{sel}}$. (iii) We highlight that Constraints (10c) and (10d) are respected by adjustments in Lines 9 and 10. By this adjustments the unit cost the bids corresponded to the saturated users and adjacent base stations approaches to infinity. (iv) Constraint (10e) is satisfied. By the termination of the while loop, we either have $\mathcal{U}^{\mathsf{sel}} = \mathcal{U}$, which means that the problem is infeasible[2], or, the second condition is violated, i.e., $D(\mathcal{I}) \leq 0$, that implies that the bids including the set $\mathcal{I}$ cover the total D2D traffic demand D. (v) Constraints (8) are automatically satisfied by the setting in Line 15. (i)-(v) prove the primal feasibility of problem sD2DAuc. □

E. Proof of Lemma 3

Proof. At τth iteration of the loop, we get

$$\begin{aligned}\hat{c}_{u,l}(\tau) &= \hat{c}_{u,l}(\tau-1) - \gamma(\mathcal{I}(\tau-1))a_{u,l}(\mathcal{I}(\tau-1)) \\ &= \hat{c}_{u,l}(\tau-2) - \gamma(\mathcal{I}(\tau-2))a_{u,l}(\mathcal{I}(\tau-2)) \\ &\quad - \gamma(\mathcal{I}(\tau-1))a_{u,l}(\mathcal{I}(\tau-1)) = \dots\end{aligned}$$

Then, by the above recursive equations, we get

$$c_{u,l} = \hat{c}_{u,l}(1) = \sum_{i=1}^{\tau} \gamma(\mathcal{I}(i))a_{u,l}(\mathcal{I}(i)) \tag{14}$$

where the first equality is the consequence of Line 6. On the other hand, we have

$$\sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \gamma(\mathcal{A})a_{u,l}(\mathcal{A}) = \sum_{i=1}^{\tau} \gamma(\mathcal{I}(i))a_{u,l}(\mathcal{I}(i)), \forall(u,l) \in \mathcal{I},$$

where τ is the iteration that the bid $(u,l) \in \mathcal{I}$ is added to set $\mathcal{I}$. This is true, since the initial values of $\gamma(\mathcal{A})$ is set to be 0 in Line 5 and just dual variables associated to the previous bids are changed in the previous iterations. Finally, by Eq. (14) and Eq. (15), we have

$$\sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \gamma(\mathcal{A})a_{u,l}(\mathcal{A}) = c_{u,l}, \forall(u,l) \in \mathcal{I}, \tag{15}$$

which proves the Lemma. □

F. Proof of Lemma 4

Proof. Indeed by the following settings Constraints (11b) and (11c) are satisfied. To proceed dual feasibility, it suffices to show that Constraint (11a) is satisfied. This would be straightforward for $(u,l) \in \mathcal{I}$, by multiplying the left-hand side of Eq. (15) by $\phi \geq 1$, so we get

$$\frac{1}{\phi}\sum_{(u,l)\in\mathcal{S}\backslash\mathcal{A}} \rho_{u,l}(\mathcal{A})a_{u,l}(\mathcal{A}) \leq c_{u,l}, \forall(u,l) \in \mathcal{I},$$

Constraint (11a) should be satisfied by the other bids, i.e., $\forall(u,l) \notin \mathcal{I}$. In this case, we define

$$\phi = \max\left\{\frac{a_{u,l}(\mathcal{A})c_{u',l'}}{a_{u',l'}(\mathcal{A})c_{u,l}}\right\}. \tag{16}$$

Then, we get

$$\frac{1}{\phi}\frac{a_{u,l}(\mathcal{A})}{c_{u,l}} \leq \frac{a_{u',l'}(\mathcal{A})}{c_{u',l'}}.$$

[2]This case in not probable in real scenarios, where usually is auction scenarios the aggregate amount of bids covered by devices is significantly higher than the covering constraint.

We can further apply the above inequality for any $(u,l) \notin \mathcal{I}$ and $(u',l') \in \mathcal{I}$ as follows

$$\frac{1}{\phi} \sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \rho_{u,l}(\mathcal{A}) \frac{a_{u,l}(\mathcal{A})}{c_{u,l}} \leq \sum_{\mathcal{A}\subseteq\mathcal{S}:(u',l')\notin\mathcal{A}} \frac{a_{u',l'}(\mathcal{A})}{c_{u',l'}} = 1,$$

so, we get

$$\frac{1}{\phi} \sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \rho_{u,l}(\mathcal{A}) a_{u,l}(\mathcal{A}) \leq c_{u,l}, \quad \forall (u,l) \notin \mathcal{I}.$$

The definition of ϕ in Eq. (16) is dependent of set $\mathcal{A}$. But, recall that $a^t_{u,l}(\mathcal{A}) = \min\{0, a^t_{u,l}, L(\mathcal{A}^t), M^t_l(\mathcal{A}^t)\}$. By checking all the possible combinations, we get

$$\frac{a_{u,l}(\mathcal{A})}{a_{u',l'}(\mathcal{A})} = \max\left\{1, \frac{a_{u,l}}{a_{u',l'}}\right\},$$

so, we have

$$\phi = \max_{u,u'\in\mathcal{U},l,l'\in\mathcal{L}} \left\{\frac{c_{u,l}}{c_{u',l'}}, \frac{c_{u,l}a_{u',l'}}{c_{u',l'}a_{u,l}}\right\}.$$

□

G. Proof of Theorem 2

Proof. First, the primal value obtained by Algorithm 1 is as follows:

$$p = \sum_{l\in\mathcal{L}} \sum_{u\in\mathcal{U}} c_{u,l} x_{u,l} = \sum_{(u,l)\in\mathcal{I}} c_{u,l}.$$

By Lemma 3, we get

$$\begin{aligned} p &= \sum_{(u,l)\in\mathcal{I}} \sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \gamma(\mathcal{A}) a_{u,l}(\mathcal{A}) \\ &= \sum_{\mathcal{A}\subseteq\mathcal{S}} \gamma(\mathcal{A}) \sum_{(u,l)\in\mathcal{I}:(u,l)\notin\mathcal{A}} a^l_{u,k}(\mathcal{A}) \\ &\leq \sum_{\mathcal{A}\subseteq\mathcal{S}} \gamma(\mathcal{A}) \Big[\sum_{(u,l)\in\mathcal{I}(\tau-1)} a_{u,l} \\ &- \sum_{(u,l)\in\mathcal{A}} a_{u,l} + a_{u',l'}(\mathcal{A}) \Big], \end{aligned} \tag{17}$$

where $\mathcal{I}(\tau-1)$ is the set of selected bids where the last bid (u',l') is excluded, i.e., $\mathcal{I}(\tau-1) = \mathcal{I}\backslash(u',l')$. By definition, $D^{\mathcal{I}(\tau-1)} = D - \sum_{(u,l)\in\mathcal{I}(\tau-1)} a_{u,l} > 0$, consequently, $\sum_{(u,l)\in\mathcal{I}(\tau-1)} a_{u,l} < D$, hence,

$$\begin{aligned} p &\leq \sum_{\mathcal{A}\subseteq\mathcal{S}} \gamma(\mathcal{A}) \Big[D - \sum_{(u,l)\in\mathcal{A}} a_{u,l} + a_{u',l'}(\mathcal{A}) \Big] \\ &\leq \sum_{\mathcal{A}\subseteq\mathcal{S}} \gamma^{\mathcal{A}} \Big[D(\mathcal{A}) + a_{u',l'}(\mathcal{A}) \Big], \end{aligned} \tag{18}$$

By definition $a_{u',l'}(\mathcal{A}) = \min\{0, a_{u',l'}, D(\mathcal{A})\} \leq D(\mathcal{A})$, then

$$p \leq \sum_{\mathcal{A}\subseteq\mathcal{S}} \gamma^{\mathcal{A}} 2D(\mathcal{A}). \tag{19}$$

After dual fitting phase in Lemma 4, we get $d = \frac{1}{\phi}\sum_{\mathcal{A}\subseteq\mathcal{S}} \gamma^{\mathcal{A}} D(\mathcal{A})$. Then, $p \leq 2\phi d$. □

H. Proof of Theorem 3

Proof. Based on the result of Lemma 5, the proof is two-fold. *First*, we begin to prove that the auction result is monotone, i.e., $\forall u \in \mathcal{U}, \forall l \in \mathcal{L}$, if $a_{u,l} = a'_{u,l}$, $c_{u,l} \leq c'_{u,l}$, and $x'_{u,l} = 1$, then $x_{u,l} = 1$. Since $a_{u,l} = a'_{u,l}$, in Line 11 of Algorithm 1, we have $\hat{a}_{u,l}(\mathcal{I}) = \hat{a}'_{u,l}(\mathcal{I})$. This is true because two bids are for the same device and the same user, hence after adjustment both amount values are the same. In addition, at beginning in Line 7 of Algorithm 1, we have $\hat{c}_{u,l} = c_{u,l} - \gamma^{\mathcal{I}} a_{u,l}(\mathcal{I})$, the second term is equal for both bids, hence since $c_{u,l} \leq c'_{u,l}$, then $\hat{c}_{u,l} \leq \hat{c}'_{u,l}$, and $\frac{\hat{c}_{u,l}}{\hat{a}_{u,l}(\mathcal{I})} \leq \frac{\hat{c}'_{u,l}}{\hat{a}'_{u,l}(\mathcal{I})}$. So, if in iteration τ of Algorithm 1 the bid with cost $c'_{u,l}$ is selected, for sure, bid with cost $c_{u,l}$ was chosen earlier in the previous iterations based on Line 12.

Second, we prove that the winners are paid threshold payment. Based on Algorithm 1, let the bid $(u', l'_u) = \arg\min_{u\in\mathcal{U}\backslash\mathcal{U}^{\mathsf{sel}}(\tau+1),l} \left\{\frac{\hat{c}_{u,l}}{\hat{a}_{u,l}(\mathcal{I}(\tau+1))}\right\}$ as the threshold bid for the selected bid $(u^\star, l^\star_u)$, i.e., (u', l'_u) is the bid that would be selected at iteration $\tau+1$ when we exclude the current selected bid $(u^\star, l^\star_u)$ at iteration τ. Based on the results in [25], it suffices to pay device $u^\star$ such that to make its normalized unit costs to be equal to the next device u', i.e., $\frac{\hat{c}_{u^\star,l^\star_u}}{a_{u^\star,l^\star_u}(\mathcal{I}(\tau))} = \frac{\hat{c}_{u',l'_u}}{a_{u',l'_u}(\mathcal{I}(\tau+1))}$. In this way, at iteration τ we have

$$\hat{c}_{u^\star,l^\star_u} = a_{u^\star,l^\star_u}(\mathcal{I}(\tau))\gamma(\mathcal{I}(\tau+1)).$$

On the other hand, we have

$$P_{u^\star} = \gamma(\mathcal{I}(\tau+1)) a_{u^\star,l^\star_u}(\mathcal{I}(\tau)) + \sum_{i=1}^{\tau-1} \gamma(\mathcal{I}(i)) a_{u^\star,l^\star_u}(\mathcal{I}(i)),$$

then, by substituting Eq. (14), we get

$$P_{u^\star} = c_{u^\star,l^\star_u} + a_{u^\star,l^\star_u}(\mathcal{I})(\gamma(\mathcal{I}(\tau+1)) - \gamma(\mathcal{I}(\tau))).$$

□

I. Proof of Lemma 7

Proof. Algorithm 1 calculates a feasible solution that satisfies the integer constraint and Constraints (2), (3), (4), and (6). Constraint (1) is also satisfied since we know that each device at the beginning of each time slot cannot submit a bid more than its residual quota, therefore Algorithm 2 respects all the constraints of problem D2DAuc and generates a feasible primal solution. □

J. Proof of Lemma 8

Proof. First, we begin the proof by constructing the dual problem. By defining dual variables exactly the same as those defined in dual problem of problem sD2DAuc and introducing dual variable κ associated to constraint (1), the dual problem is formulated as

$$\begin{aligned}
\max \quad & \sum_{\mathcal{A}\subseteq\mathcal{S}} \gamma(\mathcal{A})D(\mathcal{A}) - \sum_{u\in\mathcal{U}} \lambda_u - \sum_{l\in\mathcal{L}} \zeta^l M^l \\
& - \sum_{s\in\mathcal{S}} \mu_s C_s - \sum_{u\in\mathcal{U}} \kappa_u C_u \\
\text{s.t.} \quad & \sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \rho_{u,l}(\mathcal{A}) a_{u,l}(\mathcal{A}) + a_{u,l}\nu_{u,l} \\
& \qquad -\lambda_u - \kappa_u \leq b_{u,l}, \forall u, l \qquad (20a) \\
& \gamma(\mathcal{A}) - \rho_{u,l}(\mathcal{A}) \leq 0, \forall u, l, \mathcal{A}, \qquad (20b) \\
& \zeta^l + \sum_{s:u\in\mathcal{U}(s)} \mu_s + \nu_{u,l} \geq 0, \quad \forall u, l, \qquad (20c) \\
\text{vars.} \quad & \lambda_u \geq 0, \kappa_u \geq 0, \zeta^l \geq 0, \mu_s \geq 0, \\
& \gamma(\mathcal{A}) \geq 0, \rho_{u,l}(\mathcal{A}) \geq 0, \nu_{u,l} \geq 0.
\end{aligned}$$

Algorithm 1 generates a feasible dual solution that satisfies Constraint (11a), hence we have

$$\begin{aligned}
\textstyle\sum_{\mathcal{A}\subseteq\mathcal{S}:(u,l)\notin\mathcal{A}} \quad & \gamma(\mathcal{A}^t) a^t_{u,l}(\mathcal{A}^t) - \lambda^t_u \leq c^t_{u,l} \\
\leq \quad & b^t_{u,l} + a^t_{u,l}\kappa^{t-1}_u \leq b^t_{u,l} + a^t_{u,l}\kappa^T_u
\end{aligned}$$

Indeed, by $\nu_u = \kappa^T_u$ Constraint (20a) is satisfied. □

K. Proof of Theorem 4

Proof. Lemmas 7-8 signify that the solution calculated by Algorithm 2 is feasible. Hence, we proceed to prove competitiveness. Let define $\Delta p^t = p^t - p^{t-1}$ and $\Delta d^t = d^t - d^{t-1}$, where p^t and d^t is the objective value of problems $\mathsf{D2DAuc}$ and its dual problem calculated by Algorithm 2.

$$\begin{aligned}
\Delta p^t &= \sum_{u\in\mathcal{I}} b^t_{u,l_u} = \sum_{u\in\mathcal{I}} \left(c^t_{u,l_u} - a^t_{u,l_u}\kappa^{t-1}_u\right) \\
&= p - \sum_{u\in\mathcal{I}} a^t_{u,l_u}\kappa^{t-1}_u \\
&= p - \sum_{u\in\mathcal{I}} \left[\alpha C_u(\kappa^t_u - \kappa^{t-1}_u) - \frac{b^t_{u,l_u}}{\eta}\right] \\
&\leq \alpha d - \sum_{u\in\mathcal{I}} \left[\alpha C_u(\kappa^t_u - \kappa^{t-1}_u) - \frac{b^t_{u,l_u}}{\eta}\right] \\
&\leq \alpha\left(d - \sum_{u\in\mathcal{I}} \alpha C_u(\kappa^t_u - \kappa^{t-1}_u)\right) - \frac{\Delta p^t}{\eta}.
\end{aligned}$$

From the objective of dual problem we have $\Delta D^t = d - \sum_{u\in\mathcal{I}} \alpha C_u(\kappa^t_u - \kappa^{t-1}_u)$, hence

$$\Delta p^t \leq \alpha\Delta d^t - \frac{\Delta p^t}{\eta} \leq \frac{\alpha\eta}{\eta-1}\Delta d^t. \qquad (21)$$

As a consequence of Eq. (21), we have $p^T \leq \frac{\alpha\eta}{\eta-1} d^T$, hence Algorithm 2 is an $\frac{\alpha\eta}{\eta-1}$-approximation algorithm. Problem $\mathsf{D2DAuc}$ is the multi-slot problem and the approximation ratio achieved in Theorem 4 is the ratio of online Algorithm 2 against optimal offline solution obtained by problem $\mathsf{D2DAuc}$. In this way, $\frac{\alpha\eta}{\eta-1}$ is the competitive ratio of online Algorithm 2. □

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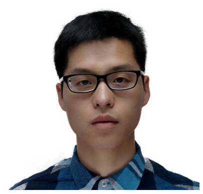

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