# Temporally Networked Cournot Platform Markets

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## Abstract

*In networked markets, information can help firms make better decisions on which market (platform), and how much, to participate. However, these markets may be temporally separated, e.g. independent system operators in different geographical locations. We model these via networked Cournot markets, but instead consider the participation of one firm to either be with the realization (or full information) of a random market, or only with the statistics of the random market, modeled by an additive zero-mean random variable $z$ on the maximal price. We show that firms not knowing the realization of the random variable $z$ would participate in both markets in the same way as if the mean was realized. We then present global effects: we prove that profit is improved for every firm when one's information improves but social welfare may get better or worst with more information.*

## 1. Introduction

Markets are increasingly spatially connected and with efficient supply chain, producers now reach far and wide to a multitude of consumers and are no longer restricted to their own local consumers. Furthermore, platforms such as eBay or Uber [1] help to connect such firms to markets in this connected system and guide consumers and producers to make efficient decisions.

Uber assign price surges in certain geographical regions to ensure that supply is given incentives to meet demand in an efficient manner [2]. Independent System Operators (ISOs) play a similar role in the power market and provide incentives to the different participants to maintain a tradeoff between socially efficient trades and stability in the system [3, 4], achieving frequency regulation and power line limits as well as maximizing social welfare.

To improve efficiency of the grid, many works suggest platforms [5] like Uber or Airbnb for the grid, where benefits include e.g. an increased participation due to small providers such as distributed energy resources (DERs) such as solar, wind, but also controllable load owners such as smart appliances. The increase in DERs would help the grid and its participants as well in terms of line management [6] (since power lines would be used less when there are more local resources) and in terms of cost, where the locational marginal prices are less likely to be increased since there is less fully utilized power lines. Another possible benefit is the increased ability to perform frequency regulation [7] using the enhanced level of controllable load participation. Other benefits include better and more competitive prices, and increased economic efficiency.

Besides being spatially (or geographically) connected, some markets are also temporally connected, and often decisions need to be made at different times (possibly due to different timezones because of difference in geographical locations) and therefore may not always be in a scenario where full information is available. An example of this is in the electricity market where a DER owner have to decide whether to supply to the current demand or at a later time or different geographical market, where the information in the later time or the later market may not be known in full. In such cases, it is important to understand how the lack of full information in later markets (or designed platforms) affects the behavior of less-informed participants, and also the other players in the system.

There are two extreme ways to model this: (i) the first is to consider the two markets as different systems, and (ii) is to assume that the firm is able to participate in both markets simultaneously with perfect information. While these two extremes represent possible scenarios, we would like our model to generalize in terms of the amount of information each firm has on the later market.

A potential resolution is to have a statistic on the later market and make an optimal decision on how to participate in the current market, given the statistic on the later market and its participants. Potentially, the more informed a firm is, the lower the variance of its corresponding statistic on the market, and the better the decision it makes. Intuitively, with greater information of the system, the participant does better. We will show that the above is true, and also, that when the participant has greater information, competition across the system improves in the right direction, and revenue of every player increases on average.

## 1.1 Networked Cournot Markets

Competition is no longer in a single, well-defined market scenario and analysis of markets have also shifted towards *networked markets*, where firms participate in multiple markets simultaneously.

One of the networked competition models that has been well studied [8, 9, 10, 11, 12] lately is the networked Cournot competition. Cournot competition, in contrast to competition by price (Bertrand Competition), competes by choosing *quantities*, and the eventual price is determined by the aggregate supply in each market.

Another motivation of networked Cournot market is that bidding processes in power markets are similar to Cournot competition and as such, the Cournot model is well used in the electricity market literature [13].

Furthermore, under mild assumptions, networked Cournot competition admits a unique Nash equilibrium [8, 9] and is an ordinal potential game in which the Nash equilibrium can be obtained via a convex optimization procedure that is polynomial-time solvable [9].

There is no consideration of the temporal factor involved in previous works, and they often assumes that price scenarios remain constant across time, and does not change. We introduce, in this work, stochasticity in the market, modeling potential lack of information for the firms able to participate in both markets.

## 1.2 Power Markets and Second Chance

Due to different timezones, power markets in distant locations within United States may close at different times. In particular, generators producing to two different markets (such as CAISO and PJM) may find it difficult to optimize over the two markets, given limited information on the eventual trading price.

By utilizing historical day ahead market bids, firms can usually obtain a good estimate of the market statistic but decisions made in the absence of full information are often suboptimal.

An informal definition for *second chance*, first introduced in [14], is when a firm is not restricted to one market, and can participate in a second or later market as an insurance for risky behavior in the first market. Second chance improves ones' profit as compared to not being able to access two markets at the same time. We model this via temporally connected markets, and whether we have full information of the later market (in which case we can effectively take part in both markets), or we do not know the realization of the second market (in which case there is no second chance and the firm optimizes over past statistics of the behavior of the later market, and is unable to use the second market as an effective source of insurance).

In this work, under stochasticity on the maximal price of the market, we show that given statistics and for the firm possibly participating in both markets, it should participate according to the mean of the random variable, as per intuition. Furthermore, we look at how second chance can improve the firm's profit, and how they affect the profit of other firms in the system.

## 1.3 Literature Review

Our work builds on, and contributes to, two related literatures: *a)* works studying platform design and *b)* works studying competition in networked markets.

**1.3.1. Platform design** The widespread success of online platforms has spurred on an increasingly growing literature focused on understanding the impacts of different designs of platforms and their success. Work in this literature has focused on a variety of topics, e.g. pricing [15], insulation [16] and competition [17] .

Our focus is on distinguishing platforms or markets based on the information they provide, and this can be either temporal as in the power market setting, or platform design in general, with the preference of withholding full information.

To this point, no analytic work has studied the impact of strategic incentives created by information or temporal interactions in online platforms. This paper provides the first such study, using the classical model of networked Cournot competition as the setting.

**1.3.2. Competition in networked settings** Networked competition appears in various forms, including networked Bertrand competition, e.g., [20, 21, 22, 23], networked Cournot competition, e.g., [8, 9, 24], and various other non-cooperative bargaining games where agents can trade via bilateral contracts and a network determines the set of feasible trades, e.g., [25, 26, 27, 28, 29].

Our paper fits squarely into the emerging literature on networked Cournot competition. The model of networked competition we study has been considered previously, beginning with [30] and continuing through [24, 8, 9, 31, 10, 11, 12]. The contribution of our work comes in two forms. Firstly, we show that withholding information in an appropriate manner can preserve competition, i.e. firms react according to the average case. If information can be withheld selectively, this may drive production rates up, which has been shown by [11, 12] to help improve efficiency in the platform. Secondly, we also show that every player gains with one player's improved information, which is a good evidence towards sharing of knowledge between players in a truthful manner.

### 1.4 Summary

We introduce a novel model in Section 2 for temporally connected networked Cournot competition, whereby participants in both markets are only given a statistic of the later market, the mean and variance of the maximal price of that market. In Section 3, we show that the optimal participation in the random market follows according to the participation with the mean of the random variable, and is independent of the variance of the random variable. We provide a motivating example in Section 4, intuiting the behavior of a single firm under second chance and provide generalizations on the result and global effects in Section 5 where there is competition, showing that the profit of both firms in the earlier and later market increase on average. Lastly, we prove in Section 6 that the firm participating in second chance always stand to gain from its participation or having increased information, and conclude in Section 8.

## 2. Model

This paper adopts the model of networked Cournot competition introduced by [8] and [9]. This model generalizes classical Cournot competition, in which firms compete in a single market.

Our interest in this work is in Cournot competition in a *networked* setting, and in addition, to consider temporal interactions between firms and markets. Specifically, we consider a set $M$ of $m$ markets and a set $F$ of $n$ firms that are connected by a set of edges $\mathcal{E} \subseteq F \times M$, where $(i, j) \in \mathcal{E}$ represents firm $i$ having access to market $j$.

For $(i, j) \in \mathcal{E}$, the *production* $q_{ij} \geq 0$ is the supply from firm $i$ to market $j$. The total production of a firm $i$ is $s_i = \sum_{j:(i,j)\in\mathcal{E}} q_{ij}$, and the total supply at market $j$ is $d_j = \sum_{i:(i,j)\in\mathcal{E}} q_{ij}$. (One can consider $q_{ij} = 0$ for all $(i, j) \notin \mathcal{E}$, and from this point on we will drop the range $(i, j) \in \mathcal{E}$ for summations.) We use $q$ to represent the vector of productions $(q_{ij})_{(i,j)\in\mathcal{E}}$. Additionally, we use $s_{-i}$ to denote the vector $(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$, and similarly for $q_{-i,j}$.

The price in a market is determined by the total supply through a demand function. Following [8], we focus on linear inverse demand throughout this paper. Specifically, the price $p_j$ at market $j$ is given by $\alpha_j - \beta_j d_j$, where $\alpha_j$ and $\beta_j$ are market specific parameters. The *demand curve* of market $j$ characterizes the price $p_j$ as a function of demand $d_j$. As in [8], a firm experiences a production cost that is quadratic in its total production, i.e., firm $i$'s cost is $c_i s_i^2$, with $c_i$ being a firm specific parameter. Note that analytical characterizations outside of linear demand and quadratic production costs are typically difficult. See [9] for a discussion.

### 2.1 Temporally Connected Markets Modeled by Stochasticity

We introduce a slight change that markets can have a stochastic $\alpha$ (maximal willingness to pay) value, introducing some stochasticity into the model:

$$p_j(d_j) = (\alpha_j + z) - \beta_j d_j$$

where $z$ is a random variable. The choice of the stochasticity on the maximal willingness to pay $\alpha$ and not the price elasticity $\beta$ is because price elasticity in a market is often kept constant while price peaks occur due to increased demand.

The assumption is that all other firms observe the realization and achieve equilibrium (existence and uniqueness of equilibrium comes from the work of [8, 9]). We limit our consideration currently to two temporal markets, one earlier and one later, with second chance being modeled via one firm's participation in both markets and knowing the realization of the random variable $z$ while not participating in second chance implies that the firm has to make a commitment to the quantity produced for the later market based on information on $z$.

## 3. Optimization under market uncertainty

When a firm is given only the statistic of the market behavior (and the information of the reported cost of the other firms) and wants to make a decision, it optimizes over the markets it participates in, knowing how its participation would change the equilibrium in each market, given the realization of random markets.

We hope that the random variable is well-behaved, where we mean that decisions can be made independent of the variance of the random variable. We restrict our consideration to the case where only one firm potentially participates in second chance, and prove the following lemma:

**Lemma 1.** *When there is only one firm participating in second chance, the solution of the optimal expected profit from a market with random* $\alpha$ *is the solution to the optimal profit from a market with the mean of* $\alpha$.

*Proof.* Note first that the quantity that the other firms produce (observing the realization) can be written in closed form as a function of the choice of the "second chance" firm's quantity to that market.

Given the "second chance" firm's fixed committed quantity to that market, the firms equivalently see a market with $\alpha^{new}(q_{sc}) = \alpha^{old} - \beta * q_{sc}$. Since each of these firms only participate in one market, the first order conditions hold and can be written:

$$\alpha_m^{new} - \beta_m S - \beta_m q_i = 2c_i q_i$$

where $S = \sum_{i=1}^{f} q_i$. and therefore, for any two firms $i,\ k$,

$$(\beta_m + 2c_i)q_i = (\beta_m + 2c_k)q_k$$

and so we can write every firms' quantity as a function of the quantity of the first firm $q_1$, i.e. $q_i = \frac{\beta_m+2c_1}{\beta_m+2c_i} q_1$, and therefore $q_i = \frac{(\beta_m+2c_i)^{-1}}{\sum_{k\in m}[(\beta_m+2c_k)^{-1}]} S$. Denote $k_i = (\beta_m + 2c_i)^{-1}$, then we have that:

$$S(q_m^{sc}) = \frac{\alpha^{new}}{\beta_m + \left(\frac{1}{\sum_j k_j}\right)}$$

where $\alpha_m^{new} = \alpha_m^{old} - \beta_m q_m^{sc}$. Note that optimizing based on the expected profit over the variable $z$, we have:

$$\begin{aligned}
\max_{q_1^{sc}, q_2^{sc}} \ & \underbrace{q_1^{sc}(\alpha_1 - \beta_1(S_1(q_1^{sc}) + q_1^{sc}))}_{\text{revenue from first market}} + \\
& + q_2^{sc}\mathbb{E}_z[\underbrace{\alpha_2 + z - \beta_2(S_z(q_2^{sc}) + q_2^{sc})}_{\text{price in market}}] \qquad (1) \\
& - c(q_1^{sc} + q_2^{sc})^2
\end{aligned}$$

where the expected price in the market is

$$\begin{aligned}
&\mathbb{E}_z[\alpha_2 + z - \beta_2(S_z(q_2^{sc}) + q_2^{sc})] \\
&= \alpha_2 - \beta_2 q_2^{sc} + \mathbb{E}_z[z - \beta_2(S_z(q_2^{sc}))] \\
&= \alpha_2 - \beta_2 q_2^{sc} + \mathbb{E}_z\left[z - \beta_2\left(\frac{\alpha_2^{new} + z}{\beta_2 + \left(\frac{1}{\sum_j k_j}\right)}\right)\right] \\
&= \alpha_2 - \beta_2 q_2^{sc} - \frac{\beta_2 \alpha_2^{new}}{\beta_2 + \left(\frac{1}{\sum_j k_j}\right)} \qquad (2) \\
&+ \mathbb{E}_z\left[z - \beta_2\left(\frac{z}{\beta_2 + \left(\frac{1}{\sum_j k_j}\right)}\right)\right] \\
&= \underbrace{\alpha_2 - \beta_2 q_2^{sc} - \frac{\beta_2 \alpha_2^{new}}{\beta_2 + \left(\frac{1}{\sum_j k_j}\right)}}_{\text{price of market with } z=0}
\end{aligned}$$

and we have that the optimal action to take is to maximize based on the mean of the random variable. Therefore, optimizing over this randomness results in reacting in a Nash sense to a game without randomness where the firm is given only the mean. □

While the proof and lemma above is for the one firm case, the proof trivially extends to the multi-firm case participating in both, where they would both still react according to the mean, even if they are given different variances.

## 4. Motivating Example: Second chance in the absence of competition

To provide intuition, we start out with the one firm, two market case, whereby the first market is deterministic and the second market is random and in this example, we assume that the random variable $z$ is bernoulli, and defined in the following manner:

$$z = \begin{cases} 1 & \text{with probability } \frac{1}{2} \\ -1 & \text{with probability } \frac{1}{2} \end{cases}$$

As we already know from the preceding section, the optimal solution with no knowledge of the realization is to allocate quantities according to the mean, i.e. $\mathbb{E}[z] = 0$.

On the other hand, for second chance, the optimal solution is with knowledge of realizations. We first compute the amount that the firm commits to the second market when there is no knowledge of the realization. Note that in general, the second chance firm contributes a non-zero aggregate supply to both markets, and that implies that all first order conditions are active. We list here some other possibilities:

1. $q_1 > 0, q_2 > 0, q_2^g > 0, q_2^b > 0$. This corresponds to the traditional case whereby participation to each market regardless of the market conditions remain active, and all first order conditions hold. In particular, the following equation, summarizing the first order conditions, hold true.

$$2c(q_1 + q_2) = \alpha_1 - 2\beta_1 q_1 = \alpha_2 - 2\beta_2 q_2$$

which gives us that aggregate supply $s$ can be written:

$$s = q_1 + q_2 = \frac{\alpha_1\beta_2 + \alpha_2\beta_1}{2(c\beta_1 + c\beta_2 + \beta_1\beta_2)}$$

which allow us to find that the equilibrium quantities at each market with no second chance will be as follow:

$$q_1 = \frac{1}{2\beta_1}(\alpha_1 - 2cs),\ q_2 = \frac{1}{2\beta_2}(\alpha_2 - 2cs)$$

with corresponding prices in each market as:

$$p_1 = \frac{1}{2}(\alpha_1 + 2cs),\ p_2 = \frac{1}{2}(\alpha_2 + 2cs),$$

Then, the profit of the firm can be written as:

$$\Pi = \frac{1}{4\beta_1}(\alpha_1^2 - 4c^2s^2) + \frac{1}{4\beta_2}(\alpha_2^2 - 4c^2s^2) - cs^2$$

Similarly, under second chance or full information, the good profit and bad profit can be written by replacing $\alpha_2$ with $\alpha_2 \pm 1$, denoted $\alpha_2^+$, $\alpha_2^-$:

$$\Pi_g = \frac{1}{4\beta_1}(\alpha_1^2 - 4c^2s_g^2) + \frac{1}{4\beta_2}[(\alpha_2^+)^2 - 4c^2s_g^2] - cs_g^2$$
$$\Pi_b = \frac{1}{4\beta_1}(\alpha_1^2 - 4c^2s_b^2) + \frac{1}{4\beta_2}[(\alpha_2^-)^2 - 4c^2s_b^2] - cs_b^2$$

where $s_g = s + \frac{\beta_1}{2(c\beta_1 + c\beta_2 + \beta_1\beta_2)}$ and $s_b = s - \frac{\beta_1}{2(c\beta_1 + c\beta_2 + \beta_1\beta_2)}$ and the profit from second chance is its expectation over all possible realizations $\Pi^{(sc)} = \frac{1}{2}(\Pi_g + \Pi_b)$, and now we show that $\Pi^{(sc)} > \Pi$, where $\Pi$ as we know from the preceding section equals that when the firm optimizes without knowledge of the realization of $z$:

$$\begin{aligned}\Pi^{(sc)} - \Pi = & \frac{1}{4\beta_1}[-4c^2(\frac{s_g^2 + s_b^2}{2} - s^2)] \\ & + \frac{1}{4\beta_2}[1 - 4c^2(\frac{s_g^2 + s_b^2}{2} - s^2)] \\ & - c(\frac{s_g^2 + s_b^2}{2} - s^2) \\ & > 0\end{aligned}$$

with $\frac{s_g^2 + s_b^2}{2} - s^2 < 0$ following from convexity.

2. $q_1 > 0, q_2 > 0, q_2^g > 0, q_2^b = 0$. In this case, when the market returns in the bad case, the firm, if participating in second chance, chooses not to participate at all. One can still follow the pipeline of the previous analysis, and find that $q_1$ is linearly decreasing in $\alpha_2$ while $q_2$ the opposite if $q_1, q_2 > 0$. However, we are discussing here a particular case where $q_2$ reduces to zero when $\alpha_2$ is small enough, say, falls below a threshold $\alpha_c$, as Fig. 1 shows.
Conspicuously, under this circumstance we have

$$\begin{aligned}\Pi^{sc} = & \frac{\Pi_g + \Pi_b}{2} = \frac{\Pi_{\alpha_2+1} + \Pi_{\alpha_2-1}}{2} = \frac{\Pi_{\alpha_2+1} + \Pi_{\alpha_c}}{2} \\ \geq & \frac{\Pi_{\alpha'} + \Pi_{\alpha_c}}{2} > \Pi_{\alpha_2} = \Pi\end{aligned}$$

where $\alpha' = 2\alpha_2 - \alpha_c$. The third equation holds as the market equilibrium (single-firm-single-market equilibrium) remains invariant for any $\alpha_2 \leq \alpha_c$. The fourth inequality is always true since $\Pi$ is monotonically increasing in $\alpha_2$. Finally the fifth inequality follows the conclusion of the previous case.

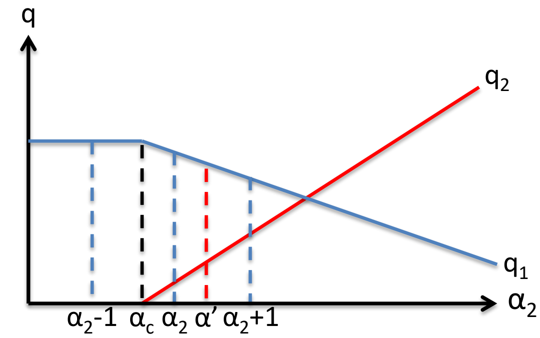


**Figure 1: Scenario 2: $q_1 > 0, q_2 > 0, q_2^g > 0, q_2^b = 0$. Second market is participated in the good setting and in the mean case but not under the bad setting.**

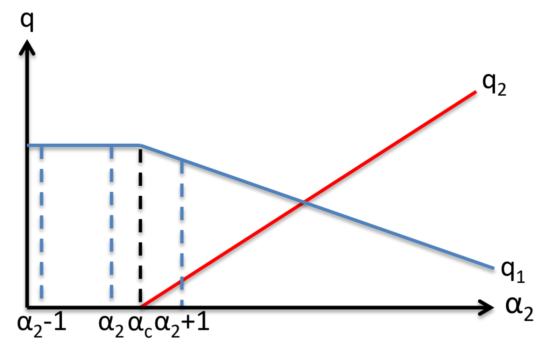


**Figure 2: Scenario 3: $q_1 > 0, q_2 = 0, q_2^g > 0, q_2^b = 0$. Second market is participated in the good setting but not in the mean case nor the bad setting.**

3. $q_1 > 0, q_2 = 0, q_2^g > 0, q_2^b = 0$. This is basically another situation that follows case 2, as illustrated in Fig. 2. With both $\alpha_2 - 1$ and $\alpha_2$, the firm only participates in the first market, i.e., $q_1 > 0, q_2 = 0, q_2^g > 0, q_2^b = 0$. Since $\Pi_b = \Pi_{\alpha_2 - 1} = \Pi_{\alpha_c} = \Pi_{\alpha_2} = \Pi$, intuitively $\Pi^{sc} = \frac{\Pi_g + \Pi_b}{2} > \Pi$.
4. $q_1 > 0, q_2 = 0, q_2^g = 0, q_2^b = 0$
The firm only participates in the first market and ignore the second one regardless of $\alpha_2$, it will neither gain nor lose.
5. $q_1 = 0, q_2 > 0, q_2^g > 0, q_2^b > 0$
The firm only participates in the second market and ignore the first one regardless of $\alpha_2$, thus

$$s = q_2 = \frac{\alpha_2}{2(c + \beta_2)}$$

with price

$$p_2 = \alpha_2 - \beta_2 s$$

We can therefore have

$$\Pi = \alpha_2 s - \beta_2 s^2 - cs^2$$

Similarly,

$$\Pi_g = (\alpha_2 + 1)s_g - \beta_2 s_g^2 - cs_g^2$$
$$\Pi_b = (\alpha_2 - 1)s_b - \beta_2 s_b^2 - cs_b^2$$

where $s_g = s + \frac{1}{2(c+\beta_2)}$ and $s_b = s - \frac{1}{2(c+\beta_2)}$. It is trivial to verify

$$\Pi^{sc} - \Pi = \frac{1}{2(c+\beta_2)} - (\beta_2 + c)\left(\frac{s_g^2 + s_b^2}{2} - s^2\right) > 0$$

since $\frac{s_g^2+s_b^2}{2} - s^2 < 0$ holds through convexity.
To sum up, excluding case 4 where we can ignore the second market a priori, profit is expected to increase and the firm stands to gain with second chance. Note that with this analysis, we also know that on average, profit increases in this non-competition case as long as $z$ is governed by some symmetric probability distribution with mean 0.

In the succeeding sections, we will make the assumption that the first order conditions hold true with equality, i.e. every link is active which implies that in every scenario, the firm would always participate in the market it is linked to.

## 5. Global Effects on other Firms

We have already noticed that participation in second chance changes the $\alpha$ value that the other firms see. In general now, we show that in markets that have firms that participate in only one market, as $\alpha$ increases, we show that profits in every firm does too. Again, while we show this for the case where only one firm potentially participates in second chance, this extends (in a tedious manner) to the case where multi firms participate in second chance together.

**Lemma 2.** *In a single market, with firms participating only in that particular market, as $\alpha$ increases, every firm's revenue increase, and vice versa. In particular, it increases as a quadratic function of the variable $\alpha$.*

*Proof.* Firstly, note from the above analysis that when every firm participate only in that market, the quantity can be written:

$$q_i = \frac{k_i}{\sum_j k_j} S$$

where $k_i = (\beta + 2c_i)^{-1}$ and

$$S = \frac{\alpha}{\beta + \frac{1}{\sum k_j}}$$

where the eventual price of the market is:

$$\begin{aligned} p(\alpha) &= \alpha - \beta S \\ &= \alpha - \frac{\alpha\beta}{\beta + \frac{1}{\sum k_j}} \\ &= \frac{\frac{\alpha}{\sum k_j}}{\beta + \frac{1}{\sum k_j}} \end{aligned}$$

We can write the revenue of the firm as a function of the maximal willingness to pay in each market $\alpha$:

$$\begin{aligned} R_i(\alpha) &= q_i \left( \frac{\frac{\alpha}{\sum k_j}}{\beta + \frac{1}{\sum k_j}} \right) \\ &= \frac{k_i}{\sum_j k_j} \left( \frac{\alpha}{\beta + \frac{1}{\sum k_j}} \right) \left( \frac{\frac{\alpha}{\sum k_j}}{\beta + \frac{1}{\sum k_j}} \right) \\ &= \frac{k_i}{(\sum_j k_j)^2} \left( \frac{\alpha}{\beta + \frac{1}{\sum k_j}} \right) \left( \frac{\alpha}{\beta + \frac{1}{\sum k_j}} \right) \\ &= \frac{\alpha^2 k_i}{(\sum_j k_j)^2 (\beta + \frac{1}{\sum k_j})^2} \end{aligned}$$

and we see that as $\alpha$ increases, the revenue at equilibrium for each firm increases, and if $\alpha$ decreases, then the revenue at equilibrium for each firm decreases. Moreover, we note that revenue of each of these firms can be written as a quadratic function of $\alpha$. □

**Lemma 3.** *Given $\alpha_1$, $\alpha_2$, the middle firm participates in each market with the corresponding quantities:*

$$q_1 = \frac{\alpha_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) - \alpha_2 \left(\frac{c}{\beta_2}\right)}{2\beta_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) + 2c}$$
$$q_2 = \frac{\alpha_2 M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) - \alpha_1 \left(\frac{c}{\beta_1}\right)}{2\beta_2 M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) + 2c}$$

*where* $M_m = \left(1 - \frac{\beta_m}{\beta_m + \left(\frac{1}{\sum_{j \in m} k_j}\right)}\right)$.

*In particular, this participation is linear in terms of $\alpha_2$, and therefore any change in $\alpha_2$ would correspondingly result in a linear change in the $\alpha_2^{new}$ seen by the other firms.*

*Proof.* When only one firm participates in second chance, then it knows exactly how the other firms will react according to the equation:

$$S_m(q_m) = \frac{\alpha_m - \beta_m q_m}{\beta_m + \left(\frac{1}{\sum_{j\in m} k_j}\right)}$$

and price at each market can correspondingly be written:

$$p_m(q_m) = \alpha_m - \beta_m S_m(q_m) - \beta q_m$$

and the optimal profit problem can be written:

$$\max_{q_1,q_2} \; q_1 p_1(q_1) + q_2 p_2(q_2) - c(q_1+q_2)^2$$

and taking derivatives with respect to $q_m$, we get:

$$2c(q_m + q_{-m}) = (\alpha_m - 2\beta_m q_m)\left(1 - \frac{\beta_m}{\beta_m + \left(\frac{1}{\sum_{j\in m} k_j}\right)}\right)$$

i.e. setting $M_m = \left(1 - \frac{\beta_m}{\beta_m + \left(\frac{1}{\sum_j k_j}\right)}\right)$, we can write:

$$q_1 = \frac{\alpha_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) - \alpha_2\left(\frac{c}{\beta_2}\right)}{2\beta_1 M_1\left(1 + \frac{c}{\beta_2 M_2}\right) + 2c} \quad (3)$$

$$q_2 = \frac{\alpha_2 M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) - \alpha_1\left(\frac{c}{\beta_1}\right)}{2\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right) + 2c} \quad (4)$$

□

## 5.1 Effects on the other participants in the later market

**Lemma 4.** *The $\alpha_2^{new}$ observed by the other firms in the second market can be written as:*

$$\alpha_2^{real} - \beta_2\left(\frac{\alpha_2^{obs} M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) - \alpha_1\left(\frac{c}{\beta_1}\right)}{2\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right) + 2c}\right)$$

*where $\alpha_2^{real}$ is the realized $\alpha_2$ and $\alpha_2^{obs}$ is what the middle firm observes, and is dependent on whether there is second chance or not. Regardless, $\alpha_2^{new}$ is a linear function of $\alpha_2$.*

*Proof.* Follows from the result in the preceding Lemma. □

When there is no second chance, the quantity that the middle firm produces for each market remains fixed as $q_m(\alpha_2)$, while if there is second chance, then the quantity produced is a function of the realized random variable $q_m(\alpha_2 + z)$.

In particular,

$$\alpha_2^{sc} = \alpha_2 + z - \beta_2\left(\frac{(\alpha_2+z) M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) - \alpha_1\left(\frac{c}{\beta_1}\right)}{2\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right) + 2c}\right),$$

and

$$\alpha_2^{nsc} = \alpha_2 + z - \beta_2\left(\frac{\alpha_2 M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) - \alpha_1\left(\frac{c}{\beta_1}\right)}{2\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right) + 2c}\right),$$

and expected revenue increase can be written as:

$$\frac{k_i}{(\sum_j k_j)^2(\beta_2 + \frac{1}{\sum k_j})^2}\mathbb{E}_z\left[(\alpha_2^{sc})^2 - (\alpha_2^{nsc})^2\right]$$

and can be simplified to be:

$$\frac{k_i}{L_2}\left[2P + P^2\right]\cdot \mathrm{Var}[z]$$

where

$$L_m = (\sum_{j\in m} k_j)^2\left(\beta_m + \frac{1}{\sum_{j\in m} k_j}\right)^2$$

and

$$P = \frac{\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right)}{2\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right) + 2c}$$

When there is second chance, $q_i$ as a function of $z$ can be written:

$$q_i^{sc} = \frac{k_i\left(\alpha_2 + z - \beta_2\left(\frac{(\alpha_2+z) M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) - \alpha_1\left(\frac{c}{\beta_1}\right)}{2\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right) + 2c}\right)\right)}{(\sum_{j\in m} k_j)\left(\beta_2 + \frac{1}{\sum_{j\in m} k_j}\right)}$$

while when there is no second chance, the participation of the middle firm becomes fixed and $q_i$ as a function of $z$ becomes:

$$q_i^{nsc} = \frac{k_i\left(\alpha_2 + z - \beta_2\left(\frac{\alpha_2 M_2 \left(1 + \frac{c}{\beta_1 M_1}\right) - \alpha_1\left(\frac{c}{\beta_1}\right)}{2\beta_2 M_2\left(1 + \frac{c}{\beta_1 M_1}\right) + 2c}\right)\right)}{(\sum_{j\in m} k_j)\left(\beta_2 + \frac{1}{\sum_{j\in m} k_j}\right)}$$

and we are interested in the quantity $\mathbb{E}[c_i(q_i^{sc})^2] - (\mathbb{E}[c_i(q_i^{nsc})^2])$:

$$\mathbb{E}[c_i(q_i^{sc})^2] - (\mathbb{E}[c_i(q_i^{nsc})^2]) = c_i\left(\frac{k_i^2(2P + P^2)}{L_2}\right)\mathrm{Var}[z]$$

and whether a profit is made depends on whether:

$$k_i - c_i k_i^2 > 0 \iff 1 > c_i k_i$$

but $c_i k_i = \frac{c_i}{\beta + 2c_i} < \frac{1}{2}$, and therefore, we obtain the following theorem:

**Theorem 5.** *A profit is always made by the other firms in the random (later) market under increased information or second chance.*

### 5.2 Effects on the other participants in the earlier market

**Lemma 6.** *The $\alpha_1^{new}$ observed by the other firms in the first market can be written as:*

$$\alpha_1 - \beta_1 \left( \frac{\alpha_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) - \alpha_2^{obs} \left(\frac{c}{\beta_2}\right)}{2\beta_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) + 2c} \right)$$

*and consequently, $\alpha_1^{new}$ is a linear function of $\alpha_2$, with a positive coefficient, indicating that as $\alpha_2$ increases (decreases), $\alpha_1^{new}$ also increases (decreases).*

*Proof.* Follows from the result in Lemma 3. □

Similar to before, the revenue of each firm participating in the first market can be written:

$$R_i(\alpha_1^{new}) = \frac{(\alpha_1^{new})^2 k_i}{(\sum_j k_j)^2 (\beta_1 + \frac{1}{\sum k_j})^2}$$

How does the random variable $z$ affect $\alpha_1^{new}$? From Lemma 5, we can see that as changes in $z$ reflect in linear changes in $\alpha_1^{new}$ in the opposite direction. Unlike the previous subsection, here we only have a indirect effect and no direct effect. When the middle firm participates in second chance, the coefficient of the quadratic term of $z$ can be written:

$$\frac{k_i}{L_1} \left[ \beta_1 \left( \frac{\frac{c}{\beta_2}}{2\beta_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) + 2c} \right) \right]^2$$

On the other hand, if the middle firm does not, then there is no effect on the earlier market, and therefore with second chance, revenue increases by:

$$\frac{k_i}{L_1} \left[ \beta_1 \left( \frac{\frac{c}{\beta_2}}{2\beta_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) + 2c} \right) \right]^2 \cdot \mathrm{Var}[z]$$

On the other hand, we can write the difference in cost:

$$\frac{c_i k_i^2}{L_1} \left[ \beta_1 \left( \frac{\frac{c}{\beta_2}}{2\beta_1 M_1 \left(1 + \frac{c}{\beta_2 M_2}\right) + 2c} \right) \right]^2 \cdot \mathrm{Var}[z]$$

and whether a profit is made depends on whether:

$$k_i - c_i k_i^2 > 0 \iff 1 > c_i k_i$$

but $c_i k_i = \frac{c_i}{\beta + 2c_i} < \frac{1}{2}$, and we get the following theorem:

**Theorem 7.** *A profit is always made by the other firms in the fixed (earlier) market under second chance*

## 6. Benefits of Participation in Second Chance

We restrict ourselves, as we did in the previous sections, in cases where only one firm has the option to participate in Second Chance. The question we want to ask in this section is whether it is always the case, as it is with the motivating examples in Section 4, that the firm with the opportunity to participate in second chance improves profit through participation. Recall from the previous section that we already have closed form solutions for the firm's participation in the two markets dependent on the whether it participates in Second Chance. Firstly, it is important to note that the eventual price of the two markets is linearly dependent of the firm's participation (since the $\alpha_m$ seen here is linearly dependent on the firm's participation), and can be written:

$$p_m(q_m) = \alpha_m - \beta_m \frac{\alpha_m - \beta_m q_m}{\beta_m + \left(\frac{1}{\sum_{j\in m} k_j}\right)} - \beta_m q_m$$

Similar to before in Section 4, we can write:

$$\delta_1 = \frac{-M_2 c}{2AB + 2Ac + 2Bc}, \; \delta_2 = \frac{M_2(A+c)}{2AB + 2Ac + 2Bc}$$

where $A = \beta_1 M_1$ and $B = \beta_2 M_2$, and again, we can have:

$$\Delta p_1 = -A\delta_1, \; \Delta p_2 = -B\delta_2 + M_2$$

and similar to before,

$$\begin{aligned}\Pi^{(sc)} - \Pi =& \Delta p_1 \delta_1 + \Delta p_2 \delta_2 - c(\delta_1 + \delta_2)^2 \\ =& D[-Ac^2 - B(A+c)^2 \\ & + 2(A+c)(AB + Ac + Bc) - cA^2] \\ =& D[(A+c)(AB + Ac + Bc)] > 0\end{aligned}$$

since $D = \frac{M_2^2}{2(AB+Ac+Bc)^2} > 0$, and therefore profit increases even in the general case, as compared to the illustration in section 4, culminating in the following theorem:

**Theorem 8.** *Every firm improves their profit when the firm exercises second chance, i.e. more information for one firm is beneficial for all parties in this system.*

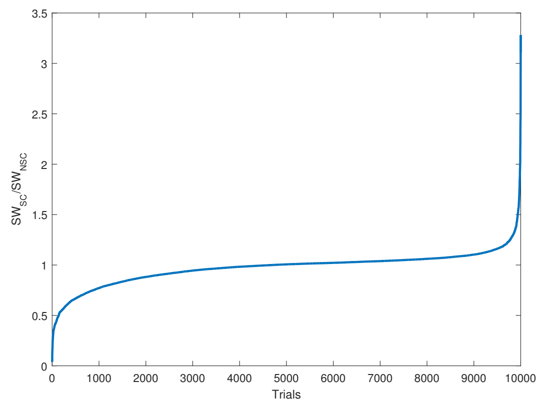


**Figure 3: Simulations illustrating the possible better and worst social welfare obtained. The results are sorted for the reader's convenience.**

## 7. Effects of Information on Social Welfare

An interesting direction for this work is whether withheld information or information asymmetry of markets can improve social welfare. We perform simulation for a small system where there is one firm potentially participating in second chance, and we are interested in the ratio between the social welfare obtained with second chance and the social welfare obtained without.

In this set of simulations, we consider the case where there are three firms in total, one firm participating in each of the two markets exclusively and one firm participating in both markets. We assume that $z$ is bernoulli as defined in section 4. Similar to our analysis, we are interested in the case where there are two markets and the choice of three firms is firstly the smallest to represent this setting and because firms in a particular knowledge group can be aggregated and seen as one firm.

Is the social welfare, alike to the individual profits, always increasing with second chance? Given results from [11], social welfare at Nash usually suffers due to decreased production levels, and information withheld may improve or degrade production levels, and therefore could potentially be used to improve social welfare.

Under 10000 simulated scenarios as described above with the parameters $\alpha$, $\beta$, and $c$ being chosen uniformly at random, we plot in Fig.3 the ratio between the average social welfare obtained with and without second chance. As anticipated, the ratio ranges from roughly $\frac{1}{4}$ to 4, signifying that the average social welfare can possibly be improved (but also possibly made worst) under information asymmetry.

## 8. Conclusion and Future Work

In this work, we introduced stochasticity in Networked Cournot Competition to take into account markets at different times, such as between geographical locations at distinct timezones or even between day-ahead or real-time participation in power markets.

With our focus on power markets, we stick to two possible markets and include stochasticity in the second (later) market. We consider one firm potentially being able to access the two markets, in two different cases, (i) the firm knows the statistics of the random variable, and (ii) the firm knows the realization of the random variable.

We find that with second chance or with precise information, not only does the firm involved increases its profit, but the other firms in the system also does on average too.

Lastly, we also show simulations that demonstrate that withholding information can make the system better or worst in terms of social welfare, and the careful design of how to and to whom to withhold information from is a good future direction to consider.

Another interesting direction to consider for future work is incorporating possibly different costs of participating in distinct markets.

## Acknowledgments

The authors would like to thank Professors Steven Low and Adam Wierman for discussions and the anonymous reviewers for their helpful suggestions.

## 9. References

[1] L. Chen, A. Mislove, and C. Wilson, "Peeking beneath the hood of uber," in *Proceedings of the 2015 ACM Conference on Internet Measurement Conference*, pp. 495–508, ACM, 2015.

[2] M. K. Chen and M. Sheldon, "Dynamic pricing in a labor market: Surge pricing and flexible work on the uber platform," tech. rep., Mimeo, UCLA, 2015.

[3] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn, "Spot pricing of electricity," 1988.

[4] G. Hamoud and I. Bradley, "Assessment of transmission congestion cost and locational marginal pricing in a competitive electricity market," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 769–775, 2004.

[5] R. Tabors, M. Caramanis, E. Ntakou, G. Parker, M. Van Alstyne, P. Centolella, and R. Hornby,

"Distributed energy resources: New markets and new products," in *Proceedings of the 50th Hawaii International Conference on System Sciences*, 2017.

[6] M. Caramanis and J. M. Foster, "Management of electric vehicle charging to mitigate renewable generation intermittency and distribution network congestion," in *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pp. 4717–4722, Ieee, 2009.

[7] J. Z. F. Pang, L. Guo, and S. H. Low, "Optimal load control for frequency regulation under limited control coverage," in *Accepted by IREP2017 Symposium, http://www.its.caltech.edu/ jzpang/IREP17freqcont.pdf*, pp. 1–7, August 2017.

[8] K. Bimpikis, S. Ehsani, and R. Ilkilic, "Cournot competition in networked markets," in *EC*, p. 733, 2014.

[9] M. Abolhassani, M. H. Bateni, M. Hajiaghayi, H. Mahini, and A. Sawant, "Network cournot competition," in *International Conference on Web and Internet Economics*, pp. 15–29, Springer, 2014.

[10] D. Cai, S. Bose, and A. Wierman, "On the role of a market maker in networked cournot competition," 2016.

[11] J. Z. Pang, H. Fu, W. Lee, and A. Wierman, "The efficiency of open access in platforms for networked cournot markets," in *IEEE International Conference on Computer Communications*, 2017.

[12] W. Lin, J. Z. Pang, E. Bitar, and A. Wierman, "Access control in online platforms: Competition and efficiency loss." Available at: `https://bitar.engineering.cornell.edu/papers/CDC17LPBW.pdf`.

[13] B. Hobbs, "Linear complementarity models of nash-cournot competition in bilateral and poolco power markets," *IEEE Transactions on power systems*, vol. 16, no. 2, pp. 194–202, 2001.

[14] P. Wang, Y. Zhang, L. Deng, M. Chen, and X. Liu, "Second chance works out better: Saving more for data center operator in open energy market," in *Information Science and Systems (CISS), 2016 Annual Conference on*, pp. 378–383, IEEE, 2016.

[15] E. G. Weyl, "A price theory of multi-sided platforms," *The American Economic Review*, vol. 100, no. 4, pp. 1642–1672, 2010.

[16] A. White and E. G. Weyl, "Insulated platform competition," *Available at SSRN 1694317*, 2016.

[17] M. Armstrong, "Competition in two-sided markets," *The RAND Journal of Economics*, vol. 37, no. 3, pp. 668–691, 2006.

[18] L. Einav, T. Kuchler, J. Levin, and N. Sundaresan, "Assessing sale strategies in online markets using matched listings," *American Economic Journal: Microeconomics*, vol. 7, no. 2, pp. 215–247, 2015.

[19] M. Dinerstein, L. Einav, J. Levin, and N. Sundaresan, "Consumer price search and platform design in internet commerce," tech. rep., National Bureau of Economic Research, 2014.

[20] C. L. Guzmán, "Price competition on network," tech. rep., 2011.

[21] E. Anshelevich and S. Sekar, "Price competition in networked markets: How do monopolies impact social welfare?," in *Web and Internet Economics*, pp. 16–30, Springer, 2015.

[22] S. Chawla and T. Roughgarden, "Bertrand competition in networks," in *Algorithmic Game Theory*, pp. 70–82, Springer, 2008.

[23] D. Acemoglu, K. Bimpikis, and A. Ozdaglar, "Price and capacity competition," *Games and Economic Behavior*, vol. 66, no. 1, pp. 1–26, 2009.

[24] R. Ilkilic, "Cournot competition on a network of markets and firms," 2009.

[25] M. Elliott, "Inefficiencies in networked markets," *American Economic Journal: Microeconomics*, vol. 7, no. 4, pp. 43–82, 2015.

[26] D. Condorelli and A. Galeotti, "Bilateral trading in networks," 2012.

[27] F. Nava, "Efficiency in decentralized oligopolistic markets," *Journal of Economic Theory*, vol. 157, pp. 315–348, 2015.

[28] D. Abreu and M. Manea, "Bargaining and efficiency in networks," *Journal of Economic Theory*, vol. 147, no. 1, pp. 43–70, 2012.

[29] M. Manea, "Bargaining in stationary networks," *The American Economic Review*, vol. 101, no. 5, pp. 2042–2080, 2011.

[30] J. I. Bulow, J. D. Geanakoplos, and P. D. Klemperer, "Multimarket oligopoly: Strategic substitutes and complements," *Journal of Political economy*, vol. 93, no. 3, pp. 488–511, 1985.

[31] S. Bose, D. W. Cai, S. Low, and A. Wierman, "The role of a market maker in networked cournot competition," in *53rd IEEE Conference on Decision and Control*, pp. 4479–4484, IEEE, 2014.