Supplementary material for "A model averaging approach for the ordered probit and nested logit models with applications".

This document provides the results of four experimental designs in addition to the two designs considered in the paper.

<u>*Design 3*</u>: The data are generated based on the ordered probit model, with $J = 3$, $p = 3$, $q = 4$, $(\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3) = (-0.075, 0.25, -0.3, 0.035, 0.01)$, X_{i1} distributed as i.i.d $N(0.5, 3)$, X_{i2} distributed as i.i.d Bernoulli(0.6), X_{i3}, Z_{i1}, Z_{i2} and Z_{i4} each distributed as i.i.d $N(0, 1)$, Z_{i3} distributed as i.i.d Bernoulli(0.4), and γ set to one of the following scenarios:

S1: $\gamma = (0.15, 0.35, 0.075, -0.02)$

S2: $\gamma = (0.15, 0.35, 0, 0)$

S3: $\gamma = (0.15, 0, 0, 0)$

<u>*Design 4*</u>: The data are generated based on the nested logit model. We let $\tau = 0.225$, and set all other parameters to the same values as in Design 3. The purpose of these two designs is to ascertain the differences in results under an increased number of explanatory variables.

<u>*Design 5*</u>: The data are generated based on the ordered probit model, with $J = 3$, $p = 1$, $q = 4$, $(\alpha_1, \alpha_2, \beta) = (-0.075, 0.25, 0.1)$, X_i, Z_{i1}, Z_{i2} and Z_{i4} each distributed as i.i.d $N(0, 1)$, Z_{i3} distributed as i.i.d Bernoulli(0.2), and γ set to one of the following scenarios:

S1: $\gamma = (0.15, 0.35, 0.075, -0.02)$

S2: $\gamma = (0.15, 0.35, 0, 0)$

S3: $\gamma = (0.15, 0, 0, 0)$

We set the number of observations of the training samples and test samples to $(n_1, n_2) = (1000, 500)$

Design 6: The data are generated based on the nested logit model. We let $\tau = 0.225$, and set all other parameters to the same values as in the Design 5. The purpose of these two designs is to ascertain the differences in results under an increased number of observations.

Each part of our simulation is based on 500 replications. We evaluate the performance of the various strategies in terms of mean squared error of forecasts (MSEF), mean absolute error of forecasts (MAEF) and hit rate (HitRate). Tables 11 - 13 below present the results.

Table 11: MSEF results

		model selection			model averaging without screening							model averaging with screening						
EXPMT	S	AIC	BIC	FIC	S-AIC	S-BIC	S-FIC	LZWZ	A-opt	JMA	EW	S-AIC	S-BIC	S-FIC	LZWZ	A-opt	JMA	EW
I		**design 1**																
	1	0.00935	$0.00988^{(-2)}$	$0.00962^{(-3)}$	0.00851	0.00883	0.00871	0.00881	0.00887	$0.00850^{(3)}$	$0.01209^{(-1)}$	0.00857	0.00885	$0.00847^{(1)}\uparrow$	$0.00871\uparrow$	$0.00866\uparrow$	0.00862	$0.00847^{(2)}\uparrow$
	2	0.00905	$0.00955^{(-2)}$	$0.00949^{(-3)}$	$0.00836^{(2)}$	0.00857	0.00867	0.00879	0.00885	$0.00829^{(1)}$	$0.01203^{(-1)}$	0.00838	0.00857	$0.00837^{(3)}\uparrow$	$0.00863\uparrow$	$0.00862\uparrow$	0.00837	$0.00838\uparrow$
	3	$0.00847^{(-2)}$	$0.00865^{(-1)}$	$0.00846^{(-3)}$	0.00764	$0.00742^{(3)}$	0.00746	0.00791	0.00824	0.00743	$0.00678^{(1)}$	0.00770	0.00753	0.00764	$0.00781\uparrow$	$0.00793\uparrow$	0.00749	$0.00736^{(2)}$
		design 2																
	1	0.00964	0.01029	$0.01198^{(-2)}$	$0.00910^{(2)}$	0.00955	0.01013	$0.01128^{(-3)}$	0.01110	0.00925	$0.01992^{(-1)}$	0.00914	0.00957	$0.00911^{(3)}\uparrow$	$0.00941\uparrow$	$0.00930\uparrow$	$0.00915\uparrow$	$0.00903^{(1)}\uparrow$
	2	0.00909	0.00956	$0.01147^{(-2)}$	$0.00881^{(3)}$	0.00905	0.00994	$0.01103^{(-3)}$	0.01087	0.00888	$0.01978^{(-1)}$	0.00883	0.00906	$0.00889\uparrow$	$0.00912\uparrow$	$0.00908\uparrow$	$0.00876^{(1)}\uparrow$	$0.00877^{(2)}\uparrow$
	3	0.00835	0.00797	$0.00904^{(-1)}$	0.00786	$0.00764^{(1)}$	0.00819	0.00852	$0.00873^{(-2)}$	0.00779	$0.00872^{(-3)}$	$0.00773^{(3)}\uparrow$	$0.00764^{(2)}$	$0.00786\uparrow$	$0.00841\uparrow$	$0.00853\uparrow$	0.00794	$0.00784\uparrow$
II		**design 1**																
	1	0.00244	$0.00281^{(-2)}$	$0.00263^{(-3)}$	$0.00225^{(1)}$	0.00257	0.00246	0.00248	0.00243	0.00230	$0.01120^{(-1)}$	$0.00227^{(2)}$	0.00258	$0.00233\uparrow$	$0.00234\uparrow$	$0.00233\uparrow$	$0.00229^{(3)}\uparrow$	$0.00257\uparrow$
	2	0.00211	0.00224	$0.00249^{(-2)}$	$0.00203^{(3)}$	0.00220	0.00239	$0.00246^{(-3)}$	0.00242	0.00204	$0.01118^{(-1)}$	$0.00202^{(1)}\uparrow$	$0.00220\uparrow$	$0.00219\uparrow$	$0.00226\uparrow$	$0.00228\uparrow$	$0.00202^{(2)}\uparrow$	$0.00236\uparrow$
	3	0.00212	0.00202	$0.00246^{(-2)}$	0.00197	0.00197	0.00229	0.00235	$0.00244^{(-3)}$	$0.00193^{(2)}$	$0.00313^{(-1)}$	$0.00191^{(1)}\uparrow$	$0.00196^{(3)}\uparrow$	$0.00205\uparrow$	$0.00227\uparrow$	$0.00232\uparrow$	0.00196	$0.00218\uparrow$
		design 2																
	1	0.00242	0.00298	$0.00389^{(-2)}$	$0.00229^{(2)}$	0.00261	0.00304	$0.00350^{(-3)}$	0.00325	0.00240	$0.02812^{(-1)}$	$0.00229^{(1)}\uparrow$	$0.00261\uparrow$	$0.00230^{(3)}\uparrow$	$0.00296\uparrow$	$0.00279\uparrow$	$0.00236\uparrow$	$0.00281\uparrow$
	2	0.00191	$0.00164^{(1)}$	$0.00304^{(-3)}$	0.00180	$0.00165^{(3)}$	0.00257	$0.00311^{(-2)}$	0.00302	0.00189	$0.02776^{(-1)}$	$0.00180\uparrow$	$0.00165^{(2)}\uparrow$	$0.00186\uparrow$	$0.00236\uparrow$	$0.00228\uparrow$	$0.00185\uparrow$	$0.00230\uparrow$
	3	0.00197	$0.00152^{(1)}$	0.00264	0.00181	$0.00153^{(3)}$	$0.00266^{(-3)}$	$0.00280^{(-2)}$	0.00250	0.00180	$0.00745^{(-1)}$	$0.00177\uparrow$	$0.00153^{(2)}\uparrow$	$0.00167\uparrow$	$0.00180\uparrow$	$0.00191\uparrow$	$0.00178\uparrow$	$0.00163\uparrow$
	(1)	0	2	0	1	1	0	0	0	1	1	3	0	1	0	0	1	1
	(2)	0	0	0	3	0	0	0	0	1	0	1	3	0	0	0	1	3
	(3)	0	0	0	2	3	0	0	0	1	0	1	1	3	0	0	1	0
	(-3)	0	0	5	0	0	1	4	1	0	1	0	0	0	0	0	0	0
	(-2)	1	3	5	0	0	0	2	1	0	0	0	0	0	0	0	0	0
	(-1)	0	1	1	0	0	0	0	0	0	10	0	0	0	0	0	0	0

[1] : (1), (2), (3), (-3), (-2), (-1) = Number of cases yielding the best, second best, third best, third worst, second worst and worst estimates respectively.

Table 12: MAEF results

		model selection			model averaging without screening							model averaging with screening						
EXPMT	S	AIC	BIC	FIC	S-AIC	S-BIC	S-FIC	LZWZ	A-opt	JMA	EW	S-AIC	S-BIC	S-FIC	LZWZ	A-opt	JMA	EW
I									**design 1**									
	1	0.11516	$0.11840^{(-2)}$	$0.11710^{(-3)}$	$0.11056^{(3)}$	0.11243	0.11195	0.11278	0.11306	0.11070	$0.13347^{(-1)}$	0.11083	0.11251	$0.11030^{(2)}\uparrow$	$0.11178\uparrow$	$0.11153\uparrow$	0.11127	$0.11029^{(1)}\uparrow$
	2	0.11352	$0.11672^{(-2)}$	$0.11658^{(-3)}$	$0.10981^{(2)}$	0.11109	0.11199	0.11287	0.11322	$0.10963^{(1)}$	$0.13339^{(-1)}$	0.10992	$0.11105\uparrow$	$0.10992\uparrow$	$0.11154\uparrow$	$0.11149\uparrow$	0.10993	$0.10988^{(3)}\uparrow$
	3	$0.11091^{(-3)}$	$0.11315^{(-1)}$	$0.11131^{(-2)}$	0.10625	0.10537	0.10531	0.10806	0.11006	$0.10497^{(2)}$	$0.10156^{(1)}$	0.10674	0.10605	0.10655	$0.10741\uparrow$	$0.10819\uparrow$	0.10525	$0.10498^{(3)}$
									design 2									
	1	0.10658	0.10896	$0.11929^{(-2)}$	$0.10412^{(2)}$	0.10630	0.11218	$0.11763^{(-3)}$	0.11683	0.10632	$0.16610^{(-1)}$	$0.10427^{(3)}$	0.10638	$0.10436\uparrow$	$0.10604\uparrow$	$0.10554\uparrow$	$0.10457\uparrow$	$0.10404^{(1)}\uparrow$
	2	0.10377	0.10527	$0.11677^{(-2)}$	$0.10269^{(3)}$	0.10386	0.11135	$0.11634^{(-3)}$	0.11576	0.10414	$0.16558^{(-1)}$	$0.10266^{(2)}\uparrow$	0.10387	$0.10336\uparrow$	$0.10461\uparrow$	$0.10442\uparrow$	$0.10242^{(1)}\uparrow$	$0.10277\uparrow$
	3	0.10185	0.09935	$0.10617^{(-2)}$	0.09994	$0.09880^{(2)}$	0.10249	0.10405	$0.10520^{(-3)}$	0.09975	$0.10639^{(-1)}$	$0.09910^{(3)}\uparrow$	$0.09872^{(1)}\uparrow$	$0.09998\uparrow$	$0.10315\uparrow$	$0.10389\uparrow$	0.10054	$0.10015\uparrow$
II									**design 1**									
	1	0.06254	$0.06488^{(-2)}$	$0.06447^{(-3)}$	$0.06001^{(1)}$	0.06294	0.06260	0.06282	0.06250	0.06069	$0.13086^{(-1)}$	$0.06014^{(2)}$	0.06296	$0.06078\uparrow$	$0.06116\uparrow$	$0.06112\uparrow$	$0.06054^{(3)}\uparrow$	$0.06293\uparrow$
	2	0.05785	$0.05681^{(3)}$	0.06234	0.05683	0.05759	0.06167	$0.06239^{(-2)}$	$0.06236^{(-3)}$	0.05705	$0.13047^{(-1)}$	$0.05663^{(1)}\uparrow$	$0.05755\uparrow$	$0.05877\uparrow$	$0.05991\uparrow$	$0.06043\uparrow$	$0.05678^{(2)}\uparrow$	$0.06034\uparrow$
	3	0.05743	$0.05330^{(1)}$	$0.06096^{(-3)}$	0.05566	$0.05404^{(3)}$	0.05983	0.06048	$0.06183^{(-2)}$	0.05513	$0.06982^{(-1)}$	$0.05441\uparrow$	$0.05392^{(2)}\uparrow$	$0.05592\uparrow$	$0.05932\uparrow$	$0.06023\uparrow$	0.05556	$0.05747\uparrow$
									design 2									
	1	0.06259	0.06910	$0.07387^{(-2)}$	$0.06094^{(3)}$	0.06485	0.06871	$0.07145^{(-3)}$	0.06962	0.06264	$0.23538^{(-1)}$	$0.06094^{(2)}\uparrow$	$0.06485\uparrow$	$0.06086^{(1)}\uparrow$	$0.06654\uparrow$	$0.06518\uparrow$	$0.06181\uparrow$	$0.06789\uparrow$
	2	0.05535	$0.05178^{(1)}$	0.06453	0.05416	$0.05189^{(3)}$	0.06305	$0.06661^{(-2)}$	$0.06632^{(-3)}$	0.05551	$0.23375^{(-1)}$	$0.05416\uparrow$	$0.05189^{(2)}\uparrow$	$0.05487\uparrow$	$0.05912\uparrow$	$0.05890\uparrow$	$0.05474\uparrow$	$0.06134\uparrow$
	3	0.05790	$0.05119^{(1)}$	0.06312	0.05581	$0.05142^{(3)}$	$0.06603^{(-3)}$	$0.06617^{(-2)}$	0.06512	0.05582	$0.11437^{(-1)}$	$0.05519\uparrow$	$0.05140^{(2)}\uparrow$	$0.05377\uparrow$	$0.05557\uparrow$	$0.05730\uparrow$	$0.05542\uparrow$	$0.05322\uparrow$
	(1)	0	3	0	1	0	0	0	0	1	1	1	1	1	0	0	1	2
	(2)	0	0	0	2	1	0	0	0	1	0	3	3	1	0	0	1	0
	(3)	0	1	0	3	3	0	0	0	0	0	2	0	0	0	0	1	2
	(-3)	1	0	4	0	0	1	3	3	0	0	0	0	0	0	0	0	0
	(-2)	0	3	5	0	0	0	3	1	0	0	0	0	0	0	0	0	0
	(-1)	0	1	0	0	0	0	0	0	0	11	0	0	0	0	0	0	0

[1] : (1), (2), (3), (-3), (-2), (-1) = Number of cases yielding the best, second best, third best, third worst, second worst and worst estimates respectively.

Table 13: HitRate results

EXPMT	S	model selection AIC	BIC	FIC	model averaging without screening S-AIC	S-BIC	S-FIC	LZWZ	A-opt	JMA	EW	model averaging with screening S-AIC	S-BIC	S-FIC	LZWZ	A-opt	JMA	EW
I									**design 1**									
	1	0.69220	$0.69170^{(-3)}$	0.69242	$0.69298^{(1)}$	$0.69168^{(-2)}$	0.69236	0.69250	0.69226	0.69274	$0.68940^{(-1)}$	$0.69294^{(2)}$	$0.69204\uparrow$	0.69222	$0.69292^{(3)}\uparrow$	$0.69234\uparrow$	0.69244	$0.69252\uparrow$
	2	0.69286	$0.69212^{(-2)}$	$0.69226^{(-3)}$	$0.69356^{(3)}$	0.69316	0.69292	0.69278	0.69280	$0.69358^{(2)}$	$0.68982^{(-1)}$	0.69318	$0.69320\uparrow$	$0.69322\uparrow$	$0.69348\uparrow$	$0.69334\uparrow$	$0.69378^{(1)}\uparrow$	$0.69336\uparrow$
	3	0.68186	0.68144	$0.68088^{(-1)}$	$0.68200^{(3)}$	0.68192	0.68190	0.68150	$0.68128^{(-2)}$	$0.68238^{(2)}$	$0.68246^{(1)}$	$0.68130^{(-3)}$	0.68184	0.68172	$0.68168\uparrow$	$0.68160\uparrow$	0.68182	0.68186
									design 2									
	1	$0.68974^{(2)}$	0.68866	$0.68792^{(-2)}$	$0.68986^{(1)}$	0.68924	0.68842	0.68830	$0.68806^{(-3)}$	0.68916	$0.67878^{(-1)}$	$0.68954^{(3)}$	0.68918	$0.68952\uparrow$	$0.68906\uparrow$	$0.68934\uparrow$	$0.68928\uparrow$	$0.68886\uparrow$
	2	$0.68868^{(3)}$	$0.68736^{(-2)}$	0.68806	0.68868	0.68808	0.68784	0.68772	0.68788	0.68844	$0.67858^{(-1)}$	0.68858	$0.68820\uparrow$	$0.68844\uparrow$	$0.68862\uparrow$	$0.68904^{(1)}\uparrow$	$0.68878^{(2)}\uparrow$	$0.68768^{(-3)}\uparrow$
	3	0.67572	0.67556	$0.67544^{(-3)}$	$0.67622^{(3)}$	0.67564	0.67584	0.67558	$0.67526^{(-2)}$	$0.67650^{(1)}$	$0.67500^{(-1)}$	0.67546	$0.67570\uparrow$	0.67558	$0.67576\uparrow$	$0.67560\uparrow$	$0.67646^{(2)}$	$0.67562\uparrow$
II									**design 1**									
	1	0.56103	$0.56057^{(-3)}$	0.56087	$0.56134^{(2)}$	0.56097	0.56106	0.56122	0.56119	0.56092	$0.55435^{(-1)}$	$0.56124^{(3)}$	$0.56099\uparrow$	0.56106	0.56110	$0.56150^{(1)}\uparrow$	$0.56097\uparrow$	$0.56049^{(-2)}\uparrow$
	2	0.55817	0.55803	$0.55758^{(-2)}$	$0.55871^{(2)}$	0.55823	$0.55789^{(-3)}$	0.55829	0.55846	0.55856	$0.55217^{(-1)}$	$0.55875^{(1)}\uparrow$	0.55820	$0.55855\uparrow$	$0.55853\uparrow$	$0.55869^{(3)}\uparrow$	0.55851	$0.55839\uparrow$
	3	0.49407	$0.49472^{(1)}$	$0.49253^{(-3)}$	0.49407	0.49413	$0.49247^{(-2)}$	0.49274	0.49277	0.49425	$0.48998^{(-1)}$	$0.49453^{(2)}\uparrow$	$0.49421\uparrow$	$0.49357\uparrow$	0.49268	$0.49291\uparrow$	$0.49437^{(3)}\uparrow$	$0.49356\uparrow$
									design 2									
	1	0.55737	0.55704	$0.55672^{(-2)}$	0.55762	$0.55765^{(3)}$	$0.55772^{(2)}$	0.55717	$0.55700^{(-3)}$	0.55721	$0.54465^{(-1)}$	0.55762	$0.55765^{(3)}$	$0.55806^{(1)}\uparrow$	0.55706	$0.55716\uparrow$	$0.55735\uparrow$	$0.55751\uparrow$
	2	0.55738	0.55781	0.55749	0.55769	$0.55800^{(1)}$	$0.55708^{(-3)}$	0.55709	$0.55685^{(-2)}$	0.55779	$0.54463^{(-1)}$	0.55769	$0.55800^{(1)}$	$0.55777\uparrow$	$0.55783\uparrow$	$0.55790^{(3)}\uparrow$	$0.55786\uparrow$	$0.55729\uparrow$
	3	0.48385	$0.48455^{(2)}$	0.48342	0.48412	0.48440	0.48336	$0.48273^{(-2)}$	$0.48318^{(-3)}$	$0.48467^{(1)}$	$0.47835^{(-1)}$	$0.48412\uparrow$	$0.48441^{(3)}\uparrow$	$0.48403\uparrow$	$0.48399\uparrow$	$0.48408\uparrow$	0.48433	$0.48429\uparrow$
	(1)	0	1	0	2	1	0	0	0	2	1	1	1	1	0	2	1	0
	(2)	1	1	0	2	0	1	0	0	2	0	2	0	0	0	0	2	0
	(3)	1	0	0	3	1	0	0	0	0	0	2	2	0	1	2	1	0
	(-3)	0	2	3	0	0	2	0	3	0	0	1	0	0	0	0	0	1
	(-2)	0	2	3	0	1	1	1	3	0	0	0	0	0	0	0	0	1
	(-1)	0	0	1	0	0	0	0	0	0	11	0	0	0	0	0	0	0

[1] : (1), (2), (3), (-3), (-2), (-1) = Number of cases yielding the best, second best, third best, third worst, second worst and worst estimates respectively.