MA3180 (Financial Mathematics I) Assignment 1

1. Sketch the graphs of the following functions.

(i) $f(x,y) = \sqrt{x^2+y^2}$.

(ii) $f(x,y) = 1 - x^2$.

(iii) $f(x,y) = 3 - x^2 - y^2$.

2. Find the indicated derivatives.

(i) $f(x) = \frac{4-x}{3+x}$, $f_x(1)$.

(ii) $f(x) = \sqrt{3-2x}$, f_x, f_{xx}.

(iii) $f(x) = x^2\sin(x^3)$, f_x.

(iv) $f(x) = (x+2)^8(x+3)^6$, $f_x(0)$, $f_{xx}(0)$.

(v) $f(x) = \sin\sqrt{x}$, f_x.

3. Fing the indicated partial derivatives.

(i) $f(x,y) = x^3y^5$; $f_x(3,-1)$.

(ii) $f(x,y) = xe^{-y} + 3y$; $f_x(1,0)$.

(iii) $z = (3xy^2 - x^4 + 1)^4$; z_x and z_y.

(iv) $z = log_x y$; z_x and z_y.

(v) $z = ln\frac{\sin x\cos y}{x^2+y^2}$; z_x.

(vi) $z = e^{xy}$; z_{xy}.

(vii) $z = x^y$; z_x and z_y.

(viii) $xy^2z^3 + x^3y^2z + x + y + z = 0$; z_x and z_y.

(ix) $xyz = \cos(x+y+z)$; z_x and z_y.

4. If f and g are twice differentiable functions of a single variable, show that the function $u(x,y) = xf(x+y) + yg(x+y)$ satifies the equation

$$u_{xx} - 2u_{xy} + u_{yy} = 0\,.$$

5. Find the tangent plane to the surface $z = \sin(x+y)$ at the point $(1,-1,0)$.

6. Use the Chain rule to find u_t and u_s.

(i) $u = 6x^3 - 3xy + 2y^2$, $x = e^t$ and $y = \cos t$.

(ii) $u = xy^2z^3$, $x = \sin t$, $y = \cos t$ and $z = 1 + e^{2t}$.

(iii) $u = xe^y + ye^{-x}$, $x = e^t$ and $y = st^2$.

7. Let $z = y^2\tan x$, $x = t^2uv$, $y = u + tv^2$; Find z_t, z_u and z_v when $t = 2$, $u = 1$ and $v = 0$.