MA3180 (Financial Mathematics I) Assignment 2

1. Find the directional derivative of the function at the given point in the direction of the vector **v**.

(i) $f(x,y) = \sqrt{x-y}$, (5,1), **v**=(12,5);

(ii) $f(x,y) = xe^{xy}$, (-3,0), **v**=(2,3);

(iii) $f(x,y,z) = xtan^{-1}(y/z)$, (1,2,-2), **v**=(1, 1, -1).

2. Find the Taylor's series of the following functions at $x = 0$.

(i) $f(x) = \sin(2x)$.

(ii) $f(x) = \frac{1}{1+x}$.

(iii) $f(x) = \sin^2 x$.

3. Find the second-order Taylor's polynomials of the following given functions and points.

(i) $f(x,y) = ln(2x+y)$, (0,1);

(ii) $f(x,y) = \sqrt{1-x^2-y^2}$, (0,0);

(iii) $f(x,y,z) = \cos(x+y+z)$, $(\pi/6, \pi/6, \pi/6)$.

4. Find the equations of the tangent plane and the normal line to the surface $4x^2 + y^2 + z^2 = 24$ at the point $(2,2,2)$.

5. If $z = f(x-y)$, show that $z_x + z_y = 0$.

6. Find the local maximum and minimum values and saddle points of the given functions

(i) $f(x,y) = xy(1-x-y)$.

(ii) $f(x,y) = x^2 + y^2 + \frac{1}{x^2y^2}$.

(iii) $f(x,y) = x^3 - 3xy + y^3$.

(iv) $f(x,y) = \frac{x^2y^2-8x+y}{xy}$.

(v) $f(x,y) = xye^{-(x^2+y^2)}$.

7. Find the shortest distance from the point $(2,-2,3)$ to the plane $6x + 4y - 3z = 2$.

8. Find the points on the surface $z^2 = xy + 1$ that are closest to the origin.

9. Use Lagrange multipliers to find the maximum values of the given function subject to the given constraint.

(i) $f(x,y) = x^2 - y^2$, $x^2 + y^2 = 1$.

(ii) $f(x,y,z) = x - y + 3z$, $x^2 + y^2 + 4z^2 = 4$.

(iii) $f(x,y,z) = x^4 + y^4 + z^4$, $x^2 + y^2 + z^2 = 1$.