MA4523 (Introduction fo Finite Elment Methods, Assignment 1

1. Show that if $w(x)$ is continuous on [0,1] and for any $v(x) \in V$,

$$\int_0^1 w(x)v(x)dx = 0$$

where

$$V = \{v : v \text{ is continuous function on } [0,1]\}$$

then $w(x) = 0$ for $x \in [0,1]$.

2. Solve the following two-point boundary value problem

$$\begin{aligned} &\tfrac{d}{dx}\left(x\tfrac{du}{dx}\right) = \tfrac{2}{x^2} \\ &u(1) = u(2) = 0 \end{aligned}$$

by using 8 linear elements with a uniform mesh.

3. Show that the problem

$$\begin{aligned} &-u'' = f(x), \qquad onI = (0,1) \\ &u(0) = u'(1) = 0 \end{aligned}$$

can be given the following variational formulation: Find $u \in V$ such that for any $v \in V$,

$$(u', v') = (f, v)$$

where $V = \{v \in H^1(I) : v(0) = 0\}$. (i) Formulate a finite element method for this problem using piecewise linear functions. (ii) Determine the corresponding linear system of equations in the case of a uniform partition. (iii) Show that the linear system has a unique solution.

4. Present the equivalent minimization problem and variational problem for the following two-point boundary value problem:

$$\begin{aligned} &-u_{xx} + u = f(x) \\ &u(0) = u(1) = 0 \end{aligned}$$

Prove the equivalence.

5. Solve the following minimization problem

$$\min_{u \in M_4^0} \frac{1}{2}\int_0^1 (u')^2\, dx - \int_0^1 u\, dx$$

where

$$M_4^0 = \{u | u \text{ polynomial of degree } \leq 4 \text{ and } u(0) = u(1) = 0\}.$$