MA4523 Assignment 3

1. Consider the problem

$$
\begin{aligned}
-\Delta u + g(x,y)u &= f \qquad in\Omega = [0,1]\times[0,1] \\
u|_{\partial\Omega} &= 0
\end{aligned}
$$

where $g(x,y) \geq 0$. Present the equivalent variational and minimization problems and prove the equivalence among these problems.

2. Solve the problem in Q1 by FEM method in the finite dimensional space

$$M_4^0 = \{u | u \text{ polynomial of degree } \leq 6 \text{ and } u_\Omega = 0\}$$

where

$$f(x,y) = (6x-2)(y^3-y^2) + (x^3-x^2)(6y-2)\,.$$

3. Consider the following biharmonic problem

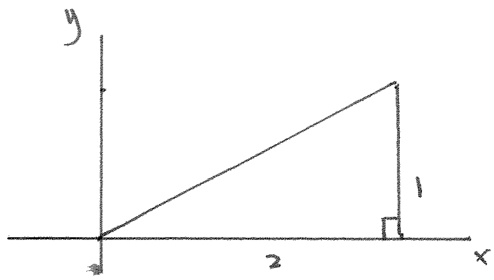
$$
\begin{aligned}
\Delta^2 u &= f(x,y), \qquad \text{in } \Omega \\
u &= \frac{\partial u}{\partial n} = 0 \qquad \text{on } \partial\Omega\,.
\end{aligned}
\tag{1}
$$

Give the equivalent variational formulation.

4. Consider the following Laplace problem:

$$
\begin{aligned}
\Delta u &= f(x,y), \qquad \text{in } \Omega \\
u &= 0 \qquad \text{on } \partial\Omega\,.
\end{aligned}
\tag{2}
$$

Find the element stiffness matrices for the following elements



5. For the problem Q4, Find all element stiffness matrices for the following geometry and assemble them into a global stiffness matrix with homogeneous boundary conditions.