

MA4523 (Introduction to FEM) Assignment 5

1. Describe the Crank-Nicolson finite difference scheme for the following initial-boundary value problem

$$\begin{aligned}u_t &= g(t)u_{xx} & x \in (0, 1), t > 0 \\u(t, 0) &= u(t, 1) = 0 \\u(0, x) &= f(x)\end{aligned}$$

2. Describe the implicit Euler scheme and Crank-Nicolson scheme for the following equation with a simple 2-D triangular linear FEM

$$\begin{aligned}u_t &= u_{xx} + u_{yy} & (x, y) \in \Omega \\u(x, y, 0) &= g(x, y) & \Omega = \{(x, y) : 0 < x, y < 1\} \\u(x, y, t) &= u(x, y, t) = 0 & \text{on } \partial\Omega,\end{aligned}\tag{1}$$

3. Prove that implicit Euler scheme for the problem (1) is unconditionally stable.
4. Analyze the stability for the following iterative scheme

$$\begin{aligned}U^{n+1} &= (I + \tau B)U^n \\U^0 &= U_0\end{aligned}$$

where

$$B = \begin{pmatrix} -1 & 1 & 0 & & & \\ 1 & -2 & 1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & -2 \end{pmatrix} \frac{1}{h^2}.$$