MA6612 (Numerical PDEs) Assignment 4

Q1. (i) Present a variational model for the following boundary value problem,

$$\begin{aligned} -\Delta u + pu &= f(x,y), \quad (x,y) \in \Omega \equiv [0,1] \times [0,1], \\ u(x,y) &= 0, \quad (x,y) \in \partial\Omega. \end{aligned} \tag{1}$$

(ii) For the case $p = 0$ and $f = x$, find a numerical solution $u_h \in V_3$ such that for any $v \in V_3$,

$$a(u_h, v) = (f, v)$$

where $a(u, v)$ is the bilinear form defined in (i) and

$$V_3 = \{u | u = \sum_{j=1} \alpha_j \phi_j,\ \phi_j = \sin(j\pi x)\sin(j\pi y)\}.$$

Q2. Present a variational model for the following biharmonic equation

$$\begin{aligned} -\Delta^2 u &= f(x,y), \quad (x,y) \in \Omega, \\ u(x,y) &= \frac{\partial u}{\partial n} = 0, \quad (x,y) \in \partial\Omega. \end{aligned} \tag{2}$$

where $\Delta^2 u = \Delta(\Delta u)$.

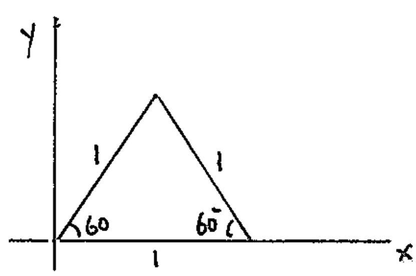
Q3. Let E be a triangle with three vertices $P_1 = (0,0)$, $P_2 = (1,0)$ and $P_3 = (1/2, \sqrt{3}/2)$. Write down these three basis functions of linear FEM on the triangle E.

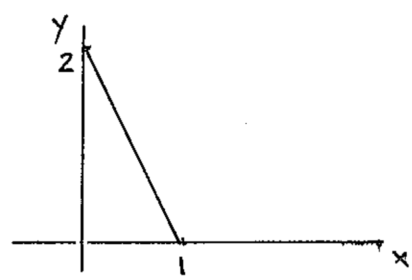
Q4. Let E be a triangle with three vertices $P_1 = (0,0)$, $P_2 = (1,0)$ and $P_3 = (0,1)$ and P_4, P_5 and P_6 be the middle points of three edges. Write down these six basis functions of quadratic FEM on the triangle E (based on the interpolation on these six points).

Q5. Consider the following Poisson's problem:

$$\begin{aligned} \Delta u = f(x,y), &\quad \text{in } \Omega \\ u = 0 &\quad \text{on } \partial\Omega. \end{aligned} \tag{3}$$

Find the element stiffness matrices for the following elements





Q6. For the problem Q5, Find all element stiffness matrices for the following geometry and assemble them into a global stiffness matrix with homogeneous Dirichlet boundary conditions.