

Assignment 1 (MA6624)

Q1. Give a formula involving a_0 , b_0 , and ϵ for the number of steps that should be taken in the bisection algorithm to ensure that the root is determined with relative precision $\leq \epsilon$. Assume $a_0 > 0$.

Q2. Write and test a subprogram to implement the bisection algorithm. Test the program on the followed function and interval:

$$2^{-x} + e^x + 2 \cos x - 6 \text{ on } [1, 3]$$

Q3.(Steffensen's method)Consider the iteration formula

$$x_{n+1} = x_n - f(x_n)/g(x_n)$$

where

$$g(x) = [f(x + f(x)) - f(x)]/f(x).$$

Show that this is quadratically convergent, under suitable hypotheses.

Q4. Consider a variation of Newton's method in which only one derivative is needed; that is,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}.$$

Find C and s such that

$$|e_{n+1}| = C|e_n|^s$$

Q5. Write a brief computer program to solve the equation $x^3 + 3x = 5x^2 + 7$ by Newton's method. Take ten steps starting at $x_0 = 5$.

Q6. Write a subprogram to carry out the secant method on a function f , assuming that two starting points are given. Test the routine on the following function

$$f(x) = e^x - \tan x$$

with $x_0 = 0.5$ and $x_1 = 1$ to get x_{10} .