Assignment #2 (MA6624)

Q1. Solve the system $Ax = b$. First, use Gaussian elimination and give the factorization $A = LU$. Second, use Gaussian elimination with scaled row pivoting.

(a)

$$A = \begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{2} \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} -1 & 1 & 0 & -3 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 3 & 0 & 1 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

Third, present the factorization $A = LDL^T$ for (b).

Q2. Let A be nonsingular and $A = LU$ be a LU-decomposition, where L is a unit lower triangular matrix. Show that the LU-decomposition is unique.

Q3. If the factor U in the LU-decomposition of A is known, what is the algorithm for calculating L?

Q4. Assume that $A = (a_{ij})$ be an $n \times n$ strictly diagonally dominant, *i.e.*,

$$\sum_{j \neq i} |a_{ij}| < |a_{ii}|, \qquad i = 1, 2, ..., n\,.$$

Then A is nonsingular.

Q5.Solve the following linear systems by the Jacobi, Gauss-Seidel and SOR ($\omega = 1.6$ and $\omega = 1.2$) methods, respectively, and present the first three iterations with the zero initial guess.

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -5 \end{bmatrix} \qquad b = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}.$$