

Assignment #3 (MA6624)

Q1. Let

$$x = \begin{bmatrix} 0.8 \\ 0.3 \\ -0.9 \\ -0.3 \end{bmatrix} \quad x = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 5 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

Calculate $\|x\|_l$ and $\|A\|_l$, $l = 1, 2, \infty$, respectively.

Q2. Let x be an m -vector and A be an $n \times m$ matrix. Show that

(i) $\|x\|_\infty \leq \|x\|_2 \leq \sqrt{m}\|x\|_\infty$,

(ii) $\frac{1}{\sqrt{m}}\|A\|_\infty \leq \|A\|_2 \leq \sqrt{n}\|A\|_\infty$.

Q3. Prove that if $\|I - A\| < 1$ and A is a square matrix, then A is nonsingular.

Q4. Prove or disprove: If $\|AB - I\| < 1$, then $\|BA - I\| < 1$.

Q5. Write a computer code for solving a tridiagonal linear system and apply your code for the system $Ax = b$ where

$$A = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & 1 \\ & & & -1 & 2 \end{pmatrix}_{n \times n} \quad b = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ n+1 \end{pmatrix} \quad (1)$$

with $n = 10$, $n = 100$ and $n = 1000$.

Q6. Solve the system in Q5 by using the LU-decomposition algorithm in Matlab.

Q7. For the linear system $Ax = b$ with

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix},$$

present Jacobi and G-S iterations and analyze the convergence of these two methods.