

Assignment #4 (MA6624)

Q1. Find the polynomials of least degree that interpolate following sets of data in Lagrange form and Newton form, respectively:

x	1	2	7	
y	2	1	146	
x	3	7	1	2
y	10	146	2	1

Q2. Let

$$L_j(x) = \prod_{i \neq j} \frac{x - x_i}{x_j - x_i}.$$

Prove

$$\sum_{j=0}^n L_j(x) = 1.$$

Q3. For the following data,

x	0.0	0.2	0.4	0.6	0.8
$f(x) = e^x$	1.00	1.22140	1.49182	1.82212	2.22554

Approximate $f(0.05)$ and $f(0.65)$ by using Lagrange polynomial of degree one and degree two, respectively.

Q4. (a). Using *plot* in MATLAB to plot $\sin(x)$ in $[0, \pi]$ with 5 equal-space points and 9 equal-space points, respectively; (b). Use *spline* in MATLAB to extend the above 5-point data to the data with 100 equal-space points and then, use *plot* (you can use any other computer software).

Q5. Generate a curve in your computer, which interpolate the data in Q3.

Q6. For $f(x) = x^m$, prove that

$$f[x_0, x_1, \dots, x_n] = \begin{cases} 1 & n = m \\ 0 & n > m \end{cases}$$

Q7. Find a quadratic polynomial $p_2(x)$ satisfying the following condition

$$p_2(x_0) = y_0, \quad p_2'(x_0) = z_0, \quad p_2(x_1) = y_1,$$

where $x_0 \neq x_1$.