Assignment #6 (MA6624)

Q1. Approximate the following integrals using midpoint rule, trapezoidal rule and Simpson's rule, respectively,

$$(a).\ \int_1^{1.5} x^2 lnx dx \qquad (b).\ \int_0^1 x^2 e^x dx\,.$$

Q2. Approximate the integrals in Q1 by using the corresponding composite rules with two subintervals.

Q3. Derive the Newton-Cotes formula for

$$\int_0^1 f(x)dx$$

based on Lagrange interpolation polynomial at the nodes $-2, -1, 0$. Apply this formula to evaluate the integral with $f = \sin \pi x$.

Q4. Determine values for α_i, $i = 1, 2, 3$i, that make the formula

$$\int_0^2 xf(x) \approx \alpha_1 f(0) + \alpha_2 f(1) + \alpha_3 f(2)$$

exact for all polynomials of degree as high as possible.

Q5. Find the three-point Gaussian quadrature rules (you can use matlab).

Q6. Use composite 2-point Gaussian quadrature rules with two subinterval to evaluate the integrals in Q1.

Q7. Use *Matlab* to evaluate the integrals in Q1.

Q8. Prove that no Gaussian quadrature rule with n nodes can be exact for all polynomials of degree $\leq 2n$.

Q9. Determine α_i and x_i for the Gaussian formula of the form

$$\int_{-1}^1 x^4 f(x) \approx \alpha_1 f(x_1) + \alpha_2 f(x_2)$$

Q10. Prove that if

$$\int_a^b \omega(x) f(x) dx = \sum_{i=1}^n \alpha_i f(x_i)$$

is exact for all polynomials of degree $\leq 2n-1$, then the polynomial $p_n(x) := \prod_{i=1}^n (x - x_i)$ is orthogonal to any polynomials of degree $\leq n-1$ on $[a, b]$ with respect to $\omega(x)$.