Vectorial Inverse Nodal Problems

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Abstract

Consider the vectorial Sturm-Liouville problem:

$$\begin{cases} -y''(x) + P(x)y(x) = \lambda y(x) \\ A_1 y(0) + A_2 y'(0) = 0 \\ B_1 y(1) + B_2 y'(1) = 0 \end{cases} :$$

where $P(x) = [p_{ij}(x)]_{i,j=1}^{d}$ is a continuous symmetric matrix-valued function defined on $[0,1]$, A_i and B_i $(i = 1,2)$ are $d \times d$ real matrices.

In a recent paper, C.L. Shen and C.T. Shieh ([?]) proved that when $d = 2$, there are infinitely many Dirichlet eigenfunctions of type (CZ) if and only if $P(x)$ is simultaneously diagonalizable. In this case, the above vectorial system can be decomposed into 2 independent scalar systems. In this paper, we extend their result to general boundary conditions. Furthermore, we prove an eigenvalue estimate for the general boundary conditions which seems to be of independent interest.

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