

Numerical Solution of Ill-Posed Cauchy Problems for Elliptic PDE's

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Abstract

We consider a two dimensional inverse steady state heat conduction problem, where the coefficients the elliptic equation depend on the solution. Cauchy data are given on part of the boundary, and we want to find another part of the boundary, whose position is unknown, but where the solution takes a certain value.

This problem occurs in an industrial application, where one wants to find the wearline of a steel furnace, by observing the temperature at points in the ceramic material of the furnace.

We solve the problem numerically as a Cauchy problem for a nonlinear elliptic PDE; i.e. we replace one of the derivatives by a bounded operator, and solve the resulting initial value problem for an ODE numerically by a standard Runge-Kutta method.

Numerical examples are given.