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New Observations in Inverse Problems and Advanced Diagnostic Analysis

by

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Abstract

Several new and enlightening scientific and mathematical observations are developed that are intended to (a) bring some closure to transient inverse analysis with respect to the development of optimal solution predictions, (b) enhance the understanding of the classical least-squares method, and (c) offer suggestions for the development of new thermal sensors. Traditional inverse analysis involves the analysis of ill-posed problems in which the numerical predictions are sensitive to input errors. Regularization methods are normally required to stabilize the prediction. Choosing the optimal regularization parameter for such methods has been the major focus for the past forty years. No steadfast mathematical rule has been established for any method used to date. The author and his coworkers have previously reported on the significance on the choice of the data used in the transient analysis. In fact, choosing the data associated with the highest temporal derivative of the differential equation describing the physics of the problem is consistent with theory.

This talk discusses numerous novel mathematical ideas associated with error analysis, attenuation/growth analysis, and further describes an experimental/analytical method for obtaining the optimal prediction based on the availability of an independently obtained secondary set of data. To illustrate the aforementioned concepts, this talk examines the first-order thermocouple compensation model expressible in the integral form as

$$T_{tc}(t) = T_o e^{-\frac{t}{\tau}} + \frac{1}{\tau}\int_{u=0}^{t} e^{-\frac{t-u}{\tau}} T_s(u)\,du; \quad t \geq 0; \tag{1}$$

where τ is the time constant. Given the device output $T_{tc}(t)$, Eq. (1) is a Volterra integral equation of the first kind for the desired surface temperature $T_s(t)$. Volterra integral equations of the first kind are known to be mildly ill-posed; that is, a small perturbation in the data may produce randomly large variations in the output. Thus, stability issues as well as numerical accuracy are germane to the resolution process. Finally, a hierarchical description is presented illustrating the correspondence between types of measurements and their influence on inverse thermal problems. Through this hierarchy, it is shown that a significant effort should be initiated to develop new sensors that are capable of estimating both the heating/cooling rate and heat flux rate. These rate measurements can be time integrated to obtain temperature and heat flux, respectively. It is shown through the hierarchy that rate measurements are necessary for assuring accuracy and stability in inverse predictions.