Wavelets and Inverse Heat Conduction

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Abstract

We consider an inverse heat conduction problem, the Sideways Heat Equation, which is a model of a problem, where one wants to determine the temperature on both sides of thick wall, but where one side is inaccessible to measurements. Mathematically it is formulated as the following problems for the heat equation in a quarter plane, with data given along the line $x = 1$, where the solution is wanted for $0 \le x \le 1$:

$$\begin{cases} u_{xx} = u_t, & x \ge 0, t \ge 0, \\ u(x,0) = 0, & x \ge 0, \\ u(1,t) = g(t), & t \ge 0, \quad u|_{x\to\infty} \text{ bounded}. \end{cases} \tag{1}$$

Of course, since g is assumed to be measured, there will be measurement errors, and we would actually have as data some function $g_\varepsilon \in L^2(\mathbb{R})$, for which

$$\|g_\varepsilon - g\|_{L^2(\mathbb{R})} = \|g_\varepsilon - u(1,\cdot)\|_{L^2(\mathbb{R})} \le \varepsilon. \tag{2}$$

where the constant $\varepsilon > 0$ represents a bound on the measurement error and we always assume that $u(x,\cdot) \equiv 0, g(\cdot) \equiv 0$ for $t \le 0$.

The problem is ill-posed in the sence that the solution (if it exists) does not depend continuously on the data. We consider regularization of this problem based on replacing the time derivative in the heat equation by wavelet-based approximations. The wavelets regularization method for this problem is proposed earlist by T.Regińska in [1] (1995), but there is a gap in the proof of main result of this paper. The present paper is firstly devoted to modify the main conclusion in [1], give a new result with complete proof. Secondly, we will present another wavelet regularization method for problem (1) by the techniques with which the non-characteristic Cauchy problem of a class of parabolic equations have been considered.[2;3]

References

[1] T.Regińska, Sideways heat equation and wavelets, J. Comput. Appl. Math, 63, 209-214(1995).

[2] Dinh Nho Hào, A mollification method for a noncharacteristic Cauchy problem for a parabolic equation, J. Math. Anal. Appl. 199, 873-909 (1996).

[3] Dinh Nho Hào, A Schneider and H-J Reinhardt, Regularization of a non-characteristic Cauchy problem for a parabolic equation, Inverse Problems. 11, 1247-1263 (1995).