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Application of Multiple-Precision Arithmetic to Some Ill-posed Problems

by

JHiroshi FUJIWARA
Kyoto University, Japan

document The aim of the presentation is to show utility of a multiple-precision arithmetic applied to numerical analysis for ill-posed problems. A new system of a multiple-precision arithmetic, which is designed by the speaker, is also introduced in the talk.

Since we admit the rounding errors in °oating point computation, we can not construct exact solutions numerically to discretized problems of functional equations, FDM or FEM schemes for PDE's. In°uence of the rounding errors does not matter in numerically stable process, but it is crucial in numerically unstable process. We hide in°uence of the rounding errors with enough digits computation under a multiple-precision arithmetic environment, and we try to construct exact numerical solutions to approximate the solutions of ill-posed problems in the sense of Hadamard.

We make a direct approach, in the present research, to numerical computations for ill-posed problems using our multiple-precision arithmetic system, and we show some numerical results. We deal with the initial value problem of the Cauchy-Riemann equation and an integral equation of the ¯rst kind with an analytic kernel function. We discretize these equations without any regularization method and construct numerical solutions using our multiple-precision arithmetic system. Our numerical results for these problems imply e®ective utility of a multiple-precision arithmetic.

In consideration of inverse problems in engineering, we need some regularization methods to reconstruct solutions from the data with the observation errors. We discuss discretization of the Tikhonov regularization and choice of the regularization parameter under multiple-precision computing. We also give an interesting comment on Hansen's L-curve analysis.

We ¯nally show the concept of the design of our multiple-precision arithmetic environment. We devise some algorithm in the package for fast computation and saving memory. We show some results of a bench mark test.