Identification of Multiple Cracks Arising in Nondestructive Testing using Superconducting Quantum Interference Device

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Abstract:

Recently, interest has grown for nondestructive testing using superconducting quantum interference device (SQUID) and several computational efforts on the related inverse problems have been accomplished for the SQUID based NDE system [1][2]. In this paper, problems on the identification of two-dimensional spatial domains arising in the detection and characterization of multiple cracks are considered. Our focusses in the identification are directed to electromagnetic inverse problems based on SQUID measurements for magnetic flux densities. To simplify the problem, we consider a two dimensional problem in which the damage is appeared on the back surface of the conducting sample. The forward analysis is considered on a cross section of the conducting material, i.e.,

$$G_c = \{ \ x = (x_1, x_2) \mid |x_1| < d, \quad 0 < x_2 < h_0, \ i = 1, 2 \ \}$$

where h_0 implies the thickness of the plate sample. Noting that the material is assumed to be of infinite extent, there is no boundary effect of the material in the x_1 direction. Hence our problem is focussed on an appropriate "window" defined by

$$G = \{ \ x \mid |x_1| < d, \quad |x_2| < h, \ h_0 < h < \infty \ \}.$$

The measurements are taken by scanning along the length of the sample on a line fixed at a center. Then the two dimensional forward problem is governed by

$$-\frac{1}{\mu_0}\left(\frac{\partial^2 A_3(x_1,x_2)}{\partial x_1^2} + \frac{\partial^2 A_3(x_1,x_2)}{\partial x_2^2}\right) + j\chi_c\sigma(x_1,x_2)\omega A_3(x_1,x_2) = \chi_c J_s(x_1,x_2) \qquad (1)$$

together with the boundary condition

$$\begin{aligned} A_3 &= 0 \quad \text{on } S_0 \\ \frac{\partial A_3}{\partial n} &= 0 \quad \text{on } S_1 \end{aligned} \qquad (2)$$

where

$$\begin{aligned} S_0 &= \{ \ x \mid |x_1| < d, \ |x_2| = h \ \} \\ S_1 &= \{ \ x \mid |x_1| = d, \ |x_2| < h \ \}. \end{aligned}$$

Our concern is to identify the number of cracks and to estimate the depth of each crack as illustrated in Fig. 1. Let M and $M_{\max}$ be the unknown number of cracks and the

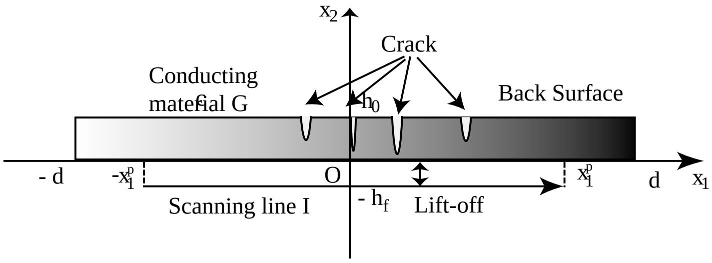

Fig. 1 Sample material with multiple cracks

maximum numbers of cracks, respectively. The admissible class is then represented by

$$\begin{aligned} C_i &= \{ \mathbf{x} \in R^2 \mid g(\mathbf{q}_i;\mathbf{x}) < 0,\ \mathbf{q}_i \in Q \subset R^2 \ \} \\ &\qquad \text{for} \quad 1 \leq i \leq M_{\max} \end{aligned} \tag{3}$$

where

$$g(\mathbf{q}_i) = a(x_1 - q_i^1)^2 + q_i^2(x_2 - h_0)^2.$$

Suppose that the multiple cracks C_i are adopted as one candidate. Then the electrical conductivities in (1) is preassigned as

$$\sigma(\mathbf{x}) = \sigma_c \quad \text{in } C_i(\mathbf{q}) \tag{4}$$

where σ_c denotes the conductivities in the crack region. For SQUID based electromagnetic nondestructive evaluation systems, observations of the magnetic flux from the front surface are represented as

$$\begin{aligned} Z_q(i) &= z(C_i(\mathbf{q}_i);x_1) \\ &= -\left(c_1\frac{\partial}{\partial x_1} + c_2\frac{\partial^2}{\partial x_1^2}\right) A_3(C_i(\mathbf{q}_i);x_1,-h_f) \quad \text{for } I. \end{aligned} \tag{5}$$

where $A_3(\cdot,x_1,-h_f)$ denotes the solution of (1) with (2). Detailed discussions on simulation results will be reported in the presentations. are used in a output least-square approach. Parameter estimation techniques based on evolutionary computation algorithm [3] are discussed and approximation schemes are developed applying a finite-element Galerkin approach. Computational techniques proposed are applied to numerical experiments to demonstrate the efficacy of the proposed schemes.

References

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