Identification and Control of Strongly Damped Nonlinear Hyperbolic Problems with Applications to Hemivariational Inequalities and Smart Materials

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Abstract

Let Ω be an open bounded subset of $\mathbb{R}^N$, V be a reflexive Banach space such that $V \hookrightarrow L^p(\Omega;\mathbb{R}^N)$ with $2 \le p < +\infty$, $H = L^2(\Omega;\mathbb{R}^N)$ and $0 < T < +\infty$. Let $j:(0,T)\times\Omega\times\mathbb{R}^N \to \mathbb{R}$, $j = j(t,x,\xi)$ be a function which is measurable in (t,x) and locally Lipschitz in ξ. We denote by $j^0(t,x,u,z)$ the Clarke directional derivative of j with respect to ξ at the point u in the direction z. Suppose that $A:[0,T]\times V \to V^*$, $A = A(t,v)$ is a nonlinear operator such that $v \mapsto A(t,v)$ is pseudomonotone on V, $B \in L(V,V^*)$ and $N \in L(V,H)$. The problem under consideration is the following. Given $u_0 \in V$, $u_1 \in H$ and $f \in L^q(0,T;V^*)$ $(1/p + 1/q = 1)$, find $u \in L^p(0,T;V)$ with $du/dt \in L^p(0,T;V)$ and $d^2u/dt^2 \in L^q(0,T;V^*)$ such that

$$
\text{(P)}\quad \begin{cases} \langle u''(t) + A(t,u'(t)) + Bu(t) - f(t), v\rangle_{V\times V^*} + \displaystyle\int_\Omega j^0(t,x,Nu(t);Nv)\,dx \ge 0 \\ \qquad \text{for a.e. } t \in (0,T) \text{ and for all } v \in V \\ u(0) = u_0, \quad u'(0) = u_1. \end{cases}
$$

The problem (P) can be written in the form of evolution inclusion

$$u''(t) + A(t,u'(t)) + Bu(t) + N^*(\partial J(t,Nu(t))) \ni f(t) \quad \text{a.e. } t \in (0,T)$$

where $J(t,v) = \int_\Omega j(t,x,v(x))\,dx$ and ∂J denotes for the Clarke generalized subdifferential of J.

The problem (P) is called (after Panagiotopoulos (1981)) a hemivariational inequality and it appears in several models of nonsmooth mechanics (nonmonotone contact problems in elasticity, skin effects of viscoelastic materials, nonconvex superpotential constitutive laws, etc.). Also, in particular, if J is differentiable, the problem (P) serves as a formulation for systems arising in "smart" materials (Banks, Smith and Wang (1996)). In this case the term $N^*(g(Nu))$ with $g(v) = J'(v)$ models a neo-Hookean type stress-strain constitutive law.

The aim of this note is twofold. First we show the (global in time) existence of weak solutions for (P). Then we deal with optimal control problems for (P). For distributed parameter control systems we establish the existence results for the Bolza control problem, the time optimal control problem and the maximum stay control problem. Independently we study the identification problem in which the control appears in the multivalued term. This situation corresponds to the inverse problem when the goal is to identify unknown real stress-strain law in materials. Finally, open problems are also outlined.