Estimation of Discontinuous Solutions of Ill-Posed Problems by Regularization for Surface Representations

Andreas Neubauer

Industrial Mathematics Institute
University of Linz
Linz, Austria

Abstract: We want to solve ill-posed (linear and nonlinear) problems $F(x) = y$ from available noisy measurements y^δ of y with $\|y^\delta - y\| \leq \delta$. We are interested in discontinuous solutions x.

Examples for such problems are, e.g., deblurring of signals and images, where F is a linear convolution integral operator, or parameter estimation problems, where $u = F(\gamma)$ solves $\mathrm{div}(\gamma \nabla u) = f$.

A serious shortcoming of standard regularization techniques is that they do not yield good results for discontinuous solutions. One method that yields good results is bounded variation regularization. A difficulty in the numerical realization od this method is that the BV-norm is not differentiable.

A new approach has been developed recently by Neubauer and Scherzer, regularization for curve representations: the solution x is interpreted as a parameterized curve $(a(t), b(t))$, with $a, b \in H^1$ and $\dot{a} \geq 0$ a.e. The advantage of this method is that the well-known results on nonlinear Tikhonov regularization in Hilbert spaces can be used to prove convergence of the regularized solutions.

This idea has been extended to two-dimensional problems by Kindermann and Neubauer, regularization for surface representation, for special parameterizations, where discontinuities are allowed only on boundaries consisting of piecewise lines that are parallel either to the x- or y-axis.

In this talk we will present new numerical results for the general case, where discontinuities may occur on arbitrary subareas. An efficient numerical realization of the general case is possible via a combination of Tikhonov regularization and moving grids.

Several numerical examples show that the new method is very efficient and yields good results.