Image Restoration by Minimizing Cost-Functions with Non-smooth Data-Fidelity Terms and Application to the Processing of Outliers

Mila Nikolova (CNRS)

Abstract.

We present a theoretical study of the recovery of an unknown image or signal $x \in \mathbb{R}^p$ from noisy data $y \in \mathbb{R}^q$ by minimizing with respect to x a regularized cost-function $J(x;y) = \Psi(x;y) + \beta\Phi(x)$, where Ψ is a data-fidelity term, Φ is a smooth regularization term and $\beta > 0$ is a parameter. Typically, $\Psi(x;y) = \|Ax - y\|^2$ where A is a linear operator. The data-fidelity terms Ψ involved in regularized cost-functions are generally smooth functions; only a few papers make an exception and they consider restricted situations. Non-smooth data-fidelity terms are avoided in image processing. In spite of this, we consider both smooth and non-smooth data-fidelity terms. Our ambition is to catch essential features exhibited by the local minimizers of regularized cost-functions in relation with the smoothness of the data-fidelity term.

In order to fix the context of our study, we consider $\Psi(x;y) = \sum_i \phi(a_i^T x - y_i)$, where a_i^T are the rows of A and ϕ is C^m on $\mathbb{R}\setminus\{0\}$. We show that if $\phi'(0^-) < \phi'(0^+)$, typical data y give rise to local minimizers $\hat{x}$ of $J(\cdot;y)$ which fit exactly a certain number of the data entries: there is a possibly large set h of indexes such that $a_i^T\hat{x} = y_i$ for every $i \in h$. In contrast, if ϕ is smooth on $\mathbb{R}$, for almost every y, the local minimizers of $J(\cdot;y)$ do not fit any entry of y. Thus, the possibility that a local minimizer fits some data entries is due to the non-smoothness of the data-fidelity term. This is a strong mathematical property which is useful in practice. By way of application, we construct a cost-function allowing aberrant data (outliers) to be detected and to be selectively smoothed. Our numerical experiments advocate the use of non-smooth data-fidelity terms in regularized cost-functions for special purposes in image and signal processing.