Direct identification of boundary values for the Lapalce equation

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Let Ω be a simply connected bounded domain in R^2 with the piecewise smooth boundary Γ. Let n be the exterior normal to the boundary. We consider the Laplace equation for unknown function u;

$$-\Delta u(x) = 0; \quad x \in \Omega \tag{1}$$

subject to the Dirichlet, Neumann, and Robin conditions;

$$u|_{\Gamma_u} = \bar{u}; \quad -\frac{\partial u}{\partial n} = q|_{\Gamma_q} = \bar{q}; \quad \text{and} \quad q|_{\Gamma_r} = \sigma(u - u_a) \tag{2}$$

given respectively on arcs Γ_u, Γ_q and Γ_r on the boundary, where $\bar{u}$ is the known $u-$value on Γ_u; $\bar{q}$ is the known value of the normal derivative on Γ_q, σ is the given positive constant on Γ_r, and u_a is the known function along Γ_r. Here we notice that the boundary components Γ_u, Γ_q, and Γ_r can be taken arbitrarily to some extent. To make our problem of eqns (??) and (??) more specific, we illustrate a sample problem. Our problem consists of finding boundary values u and q on the whole boundary Γ. In this sense we regard the problem as an inverse boundary value problem.

The authors presented a direct method for the inverse boundary value problem [?], in which only the Dirichlet and the Neumann boundary conditions are involved. In this paper, the method is extended to the problem, in which the Robin condition is also involved, in order to show that no additional difficulty will essentially arise in our approach.

This problem of finding unknown values along the whole boundary is reformulated in terms of the variational problem, which is then recast into primary and adjoint boundary value problems of the Laplace equation in the conventional forms. Identified profiles of the temperatures and heat fluxes by using the non-iterative numerical method for the sample problem are shown in the figure with reference to the corresponding direct problem.