ESTIMATION OF ALL MATHEMATICAL MODEL PARAMETERS AND EXPERIMENT INFORMATIVENESS

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This investigation examines the conditions that provide the highest achievable informativeness of experiment processing with a limited number of observation data. The following questions are raised and studied.

- How many experiments are necessary to reconstruct all properties of an object under study?
- Which conditions impede the estimation of desired quantities at any number of measurements and beforehand given high measurement precision?
- Is it possible to determine as equation coefficients and its boundary conditions simultaneously?

Our main result is given in the following theorem.

To determine all properties $\{a_k\}_{k=\overline{1;p}}$ of the object described by model

$$\sum_{k=1}^{p} a_k L_k u = f \qquad (1)$$

it is necessary and sufficient to perform a single experiment in which the object state function u does not satisfy the linear dependence condition

$$\sum_{k=1}^{p} \beta_k L_k u = 0; \quad \beta_k \in B; \qquad (2)$$

where $B = \{\beta_k = \mathrm{const}; \exists i, j \in [1;p] : \beta_i, \beta_j \neq 0\}$.

Condition (2) defines the subspace of mathematical model coefficients whose element being mapped accordingly to (1) with commuting operators retain the linear dependence condition. This results in the following corollary.

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If the solution u^* of Eq. (1) with mutually commuting operators satisfies condition (2), then the equation coefficients belong to the family

$$\beta_p a_k - \beta_k a_p = b_k, \quad k = \overline{1, p-1}; \tag{3}$$

the free term f also satisfies the condition of the linear dependence

$$\sum_{k=1}^{p} \beta_k L_k f = 0 \tag{4}$$

and the state u^* is determined as the solution of the equation

$$\sum_{k=1}^{p-1} b_k L_k u^* = \beta_p f^*$$

Theorem 1 and Corollary 1 give the complete answer to the foregoing question about the useful information contents extracted from the experiment during its processing. As is evident, a single experiment can provide simultaneous identification of all phenomenological properties of the object if the appearance of the state u^* satisfying the linear dependence condition of the initial equation terms is excluded. It is hence possible to identify the object properties starting from the observation with limited measurements. On the other hand, the results obtained attest that there exist direct problem solutions u^*, completely retained if the input function is effected according to (4) while the object properties should satisfy (3). Consequently, the variance of object properties and characteristics does not change the direct problem solution in every point of its domain.

From this viewpoint linear and nonlinear heat equations are studied. The explicit and implicit forms of unidentifiable fields are found. The results obtained show that there exist cases when every solution of the direct problem is unidentifiable state. At the same time simultaneous identification of equation coefficients and its boundary conditions is proved. The study of nonlinear heat equation permitted to find new scaling solutions.

Theoretical inferences are exemplified by the identification of all the heat object properties without numerous measurements. Simultaneous identification of thermal diffusivity, heat-transfer coefficient, initial state, axial and polar non-uniformity of temperature field are accomplished using observation data only from single sensor.

The practical meaning of the investigation consists in the determination of the conditions guaranteeing the experiment against unidentifiable states.