IDENTIFYING PARAMETERS OF LINEAR STOCHASTIC DIFFERENTIAL EQUATIONS FROM INCOMPLETE NOISY MEASUREMENTS

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Abstract. The problem taken as a whole is stated in the paper as follows: Given a system of linear stochastic differential equations with some unknown parameters and unknown stochastic inputs, determine the estimates of these parameters from the incomplete noisy measurements of the solution to the equation. The purpose of the paper is to present a possible numerical solution to this inverse mathematical problem. In the discrete time, the system is defined by the following equations:

$$
\begin{aligned}
x_{t+1} &= \Phi x_t + \Psi a_t + \Xi b_t + \Gamma c_t \\
a_{t+1} &= a_t \\
b_{t+1} &= b_t \\
c_{t+1} &= \Lambda c_t + w_t; \quad w_t \sim N(0; I)
\end{aligned}
$$

with $x \in R^n$, $a \in R^p$, $b \in R^q$, and $c \in R^r$, where the last expression, $w_t \sim N(0; I)$, stands for a random vector w_t taken from the Gaussian distribution with the mean equal to 0 (zero) and covariance equal to I (identity matrix). Parameters to be estimated are a. Parameters b are also unknown but they are not to be estimated. The only values available for producing estimates are

$$
z_t = Hx_t + v_t; \; v_t \sim N(0; R_t); \quad R_t = \mathrm{diag}\,\{r_t^{(k)}\}; k = 1; \ldots; m
$$

with the given measurement matrix $H = [h_{ij}]$.

The identification of this stochastic model is investigated as an adaptive filtering problem. The two approaches to adaptive filtering, the extended model-based approach and characteristic matching-based approach, are used together, the latter with a fictitious noise input. The fourteen heuristically formulated adaptation formulae are examined within a wide range computational experiments for adjusting the root mean square of the fictitious noise, and then the best of them is applied to solve the problem. The experiments are made on the material of one applied engineering problem, which can be stated as Inertial Navigation System Error Model Identification. All the works and experiments conducted in this paper allow us to draw the following key conclusions at this stage: (1) The most effective way to tune the fictitious noise RMS value to optimality for the extended filter-estimator is to make it proportional to a non-optimality index computed in the filter. (2) The vehicle trajectory plays an important part for success of identification in this applied inverse problem. (3) Pinpointing the vehicle-manoeuvring program of greatest practical utility can be made beforehand at the stage of simulated tests.