Some inverse problems for vectorial Sturm-Liouville equations

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Abstract

The inverse problems for Sturm-Liouville equations are almost well-studied (including inverse spectral problems and inverse nodal problems). But for the vectorial Sturm-Liouville equations, the inverse problem seems to be much more di$\pm$cult, since the equations are coupled. For example, the eigenvalues of a vetorial Sturm-Liouville equations are no more simple; two spectral data cannot determine a potential matrix. In this talk, I will discuss some direct and inverse problems for the vectorial Sturm-Liouville equation

$$y''(x) + [{}_{\,}I_{n\,i} \quad P(x)]y(x) = 0; \quad y(0) = y(1) = 0; \qquad (1)$$

where $y(x)$ is a n-dimensional vector valued function and $P(x)$ is a real symmetric matrix-valued function. We shall ¯rst study the multiplicity estimation for the eigenvalue. Then we shall show that if $Q(x)$ is even and the eigenvalues of (1) ${}_{\,}n = n$, then $Q(x)$ is uniquely determined.

References

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