Numerical Identification of the Boundary Value for the Magnetostatic Problem

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Let Ω be a bounded domain enclosed by the smooth boundary Γ in $\mathbb{R}^2$. We consider a magnetostatic problem in which the normal component of the magnetic flux density B [Wb/m^2] and the tangential component of the magnetic field H [A/m] are simultaneously prescribed on a part of the boundary $\Gamma_d \subset \Gamma$. The purpose of this study is to identify a proper boundary condition on the rest of the boundary $\Gamma_{id} := \Gamma \setminus \Gamma_d$, where any boundary conditions are not imposed, and to find a magnetic field in the domain. This problem can be regarded as an inverse boundary value problem, called magnetostatic Cauchy problem. A pair of the boundary data is said to be the Cauchy data. We define the following functional spaces:

$$H(\mathrm{rot};\Omega) := \{v \in L^2(\Omega)^2;\ \mathrm{rot}\, v \in L^2(\Omega)\};$$
$$H(\mathrm{div};\Omega) := \{v \in L^2(\Omega)^2;\ \mathrm{div}\, v \in L^2(\Omega)\}.$$

Our problem can be written as follows:

Problem 1 Find $H \in H(\mathrm{rot};\Omega) \cap H(\mathrm{div};\Omega)$ such that

$$\mathrm{rot}\, H = J \quad \text{and} \quad \mathrm{div} B = 0 \quad \text{in} \quad \Omega$$

with the boundary conditions

$$B \cdot n = \overline{B}_n \quad \text{and} \quad H \cdot \tau = \overline{H}_\tau \quad \text{on} \quad \Gamma_d;$$

where n and τ denote the unit outward normal to Γ and the unit tangent in the direction along Γ, respectively, and the Cauchy data $\overline{B}_n, \overline{H}_\tau \in H^{-1/2}(\Gamma_d)$ and the electric current density $J \in L^2(\Omega)$ [A/m^2] are given. plane.

A numerical method for the solution to this magnetostatic Cauchy problem is proposed.

The treatment is based on the method of the least squares. The steepest descent method minimizes an objective functional. Our method for the problem reduces to an iterative scheme in which primary and dual problems are alternatively solved. The mixed finite element method using the edge element solves the primary problem, and also the conventional finite element method using the nodal element solves the dual problem.

From results of numerical experiments, it is concluded that numerical solutions are in good agreement with the exact one if the boundary Γ_d, where the Cauchy data are imposed, is long. Even if the boundary Γ_d is short, good numerical solutions are obtained in the neighborhood of the boundary Γ_d.

Moreover, the case where errors are contained in the Cauchy data is considered. In such a case, a regularization term is added to the objective functional. By the similar arguments as those in the previous case where the Cauchy data are exact, a numerical algorithm is constructed. After numerical computations, the regularization term with the suitably chosen regularization parameter yields good approximate solutions for the noisy Cauchy data.