Error Estimates for Recursive Linearization of Nonlinear Ill-posed Problems

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This paper is devoted to the mathematical analysis of a general recursive linearization algorithm for solving nonlinear ill-posed problems with multiple measurements. Under some reasonable assumptions, it is shown that the algorithm is convergent with error estimates.

This work is motivated by our effort to analyze recent significant numerical results of Chen [?] for solving inverse scattering problems by recursive linearization. In [?] a recursive linearization method has been developed to solve the inverse problems when multiple measurements (frequencies or incident angles) outside the scatterer are available. The principle idea of [?], is to reduce the solution of the nonlinear inverse problem to solution of a sequence of nearly linear problems.

Let X and Y be Hilbert spaces with inner products $(\cdot;\cdot)$ and norms $\|\cdot\|$. Consider the nonlinear equation

$$F(q;t) = 0; \qquad q \in X; \qquad t \in [0;1]; \tag{1}$$

where $F: X \times [0;1] \to Y$, t is a real parameter. Assume that F is differentiable with respect to q and t. The goal is to determine a solution q of the equation

$$F(q;1) = 0. \tag{2}$$

Assume that the measurements are taken at the frequencies k_1, k_2, $\cdots$, k_N. Then the nonlinear problem (1) may be reformulated as a set of nonlinear equations:

$$\{F_{k_j}(q) = 0 \mid j = 1; \ldots; N\}. \tag{3}$$

When these nonlinear equations are solved recursively with ascending wave number, they can be reduced to a set of nearly linear problems. In this setting, the inverse problem is to determine q for each $F_{k_j}(q) = 0$. Let $\tilde{q}_{k_j}$ be the solution of $F_{k_j}(q) = 0$. Then $\tilde{q} = \lim_{N\to\infty} \tilde{q}_{k_N}$ is the solution of (2).

As an application of the convergence results, the convergence of the recursive linearization algorithm [?] is established for solving acoustic inverse scattering problems.

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