

OPTIMAL CALIBRATION OF THE LIBOR MARKET MODEL

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In this paper we claim to have developed the optimal methodology for the non-parametric calibration of market model to the benchmark at-the-money (ATM) cap/floor and swaption prices, as well as the historic correlation of the LIBOR rates. It is already known that the calibration to the correlations and the prices can be decoupled. The former requires a rank-reduction pre-processing on the historic correlation matrices, while the latter typically poses as a constrained minimization problem with quadratic objective function and nonlinear constraints. For practical use, standard methods may be too slow due to the high number of unknowns. Along the approach of Lagrange multiplier, we convert the constrained minimization problems into minimization-maximization problems (without constraint). For properly chosen objective functions, the inner maximization problems are solved with a single matrix eigenvalue decomposition. The outer minimization problems, meanwhile, are easily subdued by gradient-based descending methods due to the convexity of the objective functions. In the calibration of prices, we have used the corresponding Black's volatilities instead of prices themselves. The well-posedness of the Lagrange multiplier problems and the convergence of the descending methods are rigorously justified. Numerical results show that we have achieved very quality calibration. We have also developed a technique to calculate the hedging ratios of a derivative security with respect to the benchmark instruments, using the auxiliary results of the calibration.

Key words: LIBOR market model, non-parametric calibration, constrained optimization, Lagrange multiplier method.

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