Uniqueness in an inverse optics problem

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Abstract:

We consider the scattering by the perfectly reflecting periodic structure and we discuss the two dimensional modelling. Let $f \in C^2(\mathbb{R})$ be 2π-periodic, $f(x) < 0$ for $x \in \mathbb{R}$. We set

$$\Omega_f = \{(x,y);\, y > f(x),\ x \in \mathbb{R}\}.$$

Then we regard $\partial\Omega_f = \{(x,y);\, y = f(x),\ x \in \mathbb{R}\}$ as a periodic interface which we should determine by scattering data. For this, we introduce an incident field $u^I(x,y;k)$ given by

$$u^I(x,y;k) = \exp\{ik(x\sin\theta - y\cos\theta)\},$$

which is a time-harmonic electromagnetic plane wave, and θ is considered as incident angle.

Then the resulting scattering field $u^S(x,y;k)$ satisfies the Helmholtz equation (1) with the perfect reflection boundary condition (2) and the outgoing wave condition (3):

$$\Delta u^S + k^2 u^S = 0 \qquad in \quad \Omega_f.$$

$$u^S + u^I = 0 \qquad on \quad \partial\Omega_f.$$

u^S satisfies the outgoing wave condition: $u^S(x,y) = \sum_{n\in\mathbb{Z}} u_n e^{i(\alpha_n x + \beta_n y)}$ if $y > \|f\|_{C[0,2\pi]}$.

Here and henceforth, we set

$$\left\{\alpha_n = n + k\sin\theta,\ \beta_n = \sqrt{k^2 - (n + k\sin\theta)^2}, \quad 0 \le \arg\beta_n < \pi.\right.$$

Moreover we pose the $(k\sin\theta)$-quasi-periodicity condition for u^S:

$$u^S(x + 2\pi, y; k) = \exp(2\pi i k\sin\theta)\, u^S(x,y;k)$$

for all $(x,y) \in \mathbb{R}^2$.

We consider an inverse optics problem.

Inverse Problem of Diffractive Optics

Determine $y = f(x)$, $x \in \mathbb{R}$, from measurement

$$u^S(x, 0; k), \quad x \in (0, 2\pi),$$

where u^S satisfies (1) - (4).

For this inverse problem, the uniqueness is proved for a lossy medium (i.e., $\mathrm{Im} k > 0$) by Bao [1], and for the case of $k \in \mathbb{R}$ by Hettlich and Kirsch [2] where one has to measure $u^S(x, 0; k_j)$, $1 \le j \le N$, after k_j, $1 \le j \le N$, are suitably chosen. We will show new types of uniqueness results: