AN INVERSE PROBLEM FOR THE WAVE EQUATION

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Consider the following Cauchy problem for the wave equation

$$\left.\begin{aligned} & u_{xx} - u_{tt} = \pm q_1(x)u_t - q_0(x)u; \quad -\infty < x < \infty; \ t > 0; \\ & u_{|t=0} = 0; \quad u_{t|t=0} = 2\delta(x); \end{aligned}\right\} \tag{1}$$

where $\delta(x)$ is the Dirac delta-function, $q_j(x)$ are complex-valued functions, $q_j(-x) = q_j(x)$; $q_j(x) \in W_1^j(0,\infty)$: Denote $r^\pm(t) := u^\pm(0;t)$; where $u^\pm(x;t)$ is the solution of (1).

We study the inverse problem of recovering coefficients of equation (1) from the given functions $r^\pm(t)$.

We note that this inverse problem is equivalent to the inverse spectral problem of recovering the coefficients of non-selfadjoint pencil

$$y'' + (\rho^2 + i\rho q_1(x) + q_0(x))y = 0; \quad x > 0 \tag{2}$$

from the given Weyl function $M(\rho) := \Phi(0;\rho)$; where $\Phi(x;\rho)$ is the solution (2) under the conditions $\Phi'(0;\rho) = 1$; $\Phi(x;\rho) = O(\exp(\pm i\rho x))$; $x \to \infty$; $\pm \mathrm{Im}\,\rho > 0$:

Theorem. The specification of $r^\pm(t)$; $t \geq 0$; uniquely determines the functions $q_1(x)$ and $q_0(x)$; $x \geq 0$:

Using the method of spectral mappings (see [1]) we also obtain a constructive procedure for calculating the global solution of the inverse problem considered along with necessary and sufficient conditions for its global solvability. We note that in [2] local solution of the inverse problem for equation (1) is obtained in a neighbourhood of the origin.

Remark. The specification of only one of the functions r^+ (or r^-) is not sufficient for the unique determination of the function q_1 and q_0: However the specification r^+ (or r^-) uniquely

determines one of the coefficients q_1 (or q_0) provided that the second one is known a priori.

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References

[1] Yurko V.A., Inverse Spectral Problems for Differential Operators and their Applications. Gordon and Breach, New York, 2000.

[2] Romanov V.G. and Kabanikhin S.I., Inverse problems for Maxwell's equations. Inverse and Ill-posed Problems Series. VSP, Utrecht , 1994.