An Inverse Problem of Derivative Security Pricing

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Abstract

In this article an inverse problem on derivative security of interest rate is discussed. Suppose that the short term interest rate r follows a random walk(ITO Process)

$$dr = \mu(r)dt + w(r)dz;$$

where dz is a normally distributed random varible with zero mean and variance dt, $r \in [0;R]$,mean reverting function $\mu(r)$ and volatility $w(r)$ are sufficiently differentiable functions, satifying

$$\mu(0) \geq 0; \quad \mu(R) \leq 0; \quad \text{and } w(0) = w(R) = 0:$$

$V(t;r;T)$, price of zero-coupon bond, satisfies the following partial differential equation

$$\frac{\partial V}{\partial t} + \frac{w^2(r)}{2}\frac{\partial^2 V}{\partial r^2} + (\mu(r) + \lambda(t)w(r))\frac{\partial V}{\partial r} - rV = 0$$

where T is maturity, t is time, $\lambda(t) = \frac{\mu - r}{\sigma}$ is risk maket price of interest rate r, μ and σ are the expected return and volatility of the derivative security of the interest rate. The final condition is given by

$$V(T;r;T) = Z; \quad 0 < T \leq T_{max};$$

where Z is face value of bond.

The inverse problem discussed in the article is to determine the risk market price of interest rate $\lambda(t)$ from the specified current market prices

$$V(t = 0;r_0;T) = V(T); \quad 0 < T \leq T_{max}:$$

The inverse problem for $\lambda(t)$ is transformed into the system of adjoint equation

$$\frac{\partial U}{\partial t} - \frac{1}{2}\frac{\partial^2}{\partial r^2}(w^2(r)U) + \frac{\partial}{\partial r}((\mu(r) + \lambda(t)w(r))U) + rU = 0;$$
$$(t;r) \in (0;\infty) \times [0;R];$$

with boundary conditions

$$U(t;0) = U(t;R) = 0;$$

and initial condition

$$U(t = 0;r) = \delta(r - r_0);$$

where $\delta(r - r_0)$ is Delta function, and the integral equation with current market price data

$$\lambda(T)\int_0^R w(r)U(T;r)dr + \int_0^R (\mu(r) - r^2)U(T;r)dr = -\frac{V''(T)}{Z}$$

The integral equation is nonlinear,because $U(T;r)$ depends on $\lambda(t)$ in itself.

The numerical algorithm to solve this system is constructed and some numerical experiments for the data without error and with random error are performed. The numerical results show that the algorithm is quite efficient and robust.